Homework 6: §2.3: 2, 5, 6, 7, 10

2. (a) Suppose there is some nontrivial relation aRb, then by symmetry bRa, but by antisymmetry NOT bRa, thus deriving a contradition.

(b) By definition, if R is antisymmetric, then when aRb, bRa never holds. Therefore, the statement "Whenever aRb and bRa, we have $a = b$" holds trivially.

(c) The relation "=" on the set $\{1, 2\}$.

(d) Denote by R^c the complement of a symmetric relation R. Suppose that R^c were not symmetric. That is, there exist an a and b such that aR^cb but not bR^ca. Then for the original relation R we must have bRa but not aRb. This is a contradiction, as R was assumed to be symmetric.

(e) Suppose R is transitive, and R^{rev} is its reverse. Now suppose $aR^{rev}b$ and $bR^{rev}c$, then correspondingly we must have bRa and cRb. By the transitivity of R we have cRa, and therefore $aR^{rev}c$. Thus, by definition, R^{rev} is also transitive.

(f) Take $X := \{1, 2\}$ and the relation $R := \{(2, 2)\} \subset X \times X$. Then R is symmetric and trivially transitive, but is not reflexive.

5. Observe that R is reflexive, as it contains the diagonal. Transitivity and weak antisymmetry may be verified by observing that R admits a valid Hasse diagram, as displayed in the back of the book.

6. See back of book for Hasse diagram.

7. Essentially one must check symmetry, reflexivity, and transitivity by brute force. However, there are some short ways of describing this: Observe that R contains the diagonal, and so is reflexive. The non-diagonal elements of R are $\{(1,2),(2,1),(3,4),(4,3)\}$. As this set is "symmetric about the diagonal", R is symmetric. Transitivity may be verified by noting if aRb and bRc, then a, b, and c cannot all be distinct, so transitivity is "trivial" in some sense.

10. If matrices A, B are equivalent, ie: $A = P^{-1}BP$ for an invertible matrix P, then by letting $Q = P$, we see that $A = P^{-1}BQ$ for this choice of Q. Therefore A and B are simmilar.