PRACTICE MIDTERM FOR MAT 312

(1) Let $C \subset Z_2^{20}$ denote a set of code words. Suppose that $d = 7$ for this code.

(a) How many transmission errors can be detected by this code? Explain why.
(b) How many transmission errors can be corrected by this code? Explain why.
(c) Give a simple example of a code C with $d = 7$.

(2) Let H denote a 4×6 binary matrix, and let $C \subset Z_2^6$ denote all the binary 6-tuples $\mathbf{c}$ such that $H\mathbf{c}^t = \mathbf{0}$.

(a) Explain why C is a group code.

In (b)-(e) below assume that H is equal to

$$\begin{matrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{matrix}$$

Try do (b)-(d) without listing the code words in C.

(b) Show that C is single error detecting and single error correcting.
(c) Compute d for this code.
(d) A code word $\mathbf{c}$ is transmitted and a binary 6-tuple $\mathbf{r}$ is received. If $\mathbf{r} = 110000$ then compute the syndrome of $\mathbf{r}$; if at most one transmission error has been made, then find $\mathbf{c}$.
(e) List all the code words in C.

(3) Let G denote a group having the just the 4 elements $G = \{a, b, c, d\}$. Suppose that $ab = a$ and $a^3 = c$.

(a) Which element of G is the identity.
(b) Fill in the multiplication table for G.
(c) Is G isomorphic to Z_4? Is G isomorphic to $Z_2 \times Z_2$?

(4) Explain why each of the following pairs of groups are (or are not) isomorphic.

(a) Z_6 and $Z_2 \times Z_3$.
(b) S_3 and Z_6.
(c) S_4 and S_3.
(d) S_2 and Z_2.

(5) Consider the permutation $\sigma \in S_8$ defined by $\sigma = (126)(8241)(5368)$.

(a) Write σ as a 2×8 matix having

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

for first row.

(b) Write σ^{-1} as a 2×8 matrix (as in part (a)).

(c) Compute $order(\sigma)$.

(d) Write σ as a product of disjoint cycles.