MAT 310: HW5

(1) Let V and W denote n-dimensional vector spaces over the field F, and let $V' \subset V$ and $W' \subset W$ denote m-dimensional subspaces of V and W respectively. Show that there is an isomorphism $T : V \longrightarrow W$ such that $T(V') = W'$.

(2) Do problems #2,3,4,7,16 in section 2.4.

(3) Let V, W, X, Y denote vector spaces over the field F, and let $T : V \longrightarrow W$ and $S : X \longrightarrow Y$ denote linear transformations. We say that T is *isomorphic* to S if there exits two isomorphisms $f : V \longrightarrow X$ and $g : W \longrightarrow Y$ such that $g \circ T = S \circ f$.

(a) Show that if T is isomorphic to S then S is isomorphic to T.
(b) Show that T is isomorphic to S iff the following hold: $dim(V) = dim(X)$; $dim(W) = dim(Y)$; $nullity(T) = nullity(S)$.
(c) Show that T is isomorphic to S iff there are basis $\alpha, \beta, \sigma, \tau$ for V, W, X, Y respectively such that

$$[T]_{\alpha}^{\beta} = [S]_{\sigma}^{\tau}.$$

(4) Do problems #2bc,3cd,4,5,6b,8,10 in section 2.5.