(1) Let $P(F)$ denote the vector space of all polynomials in the one variable $t$ with coefficients in the field $F$.

(a) For $f(t), g(t), h(t) \in P(F)$, if $f(t)h(t) = g(t)h(t)$ then show that $f(t) = g(t)$.

(b) For $f(t), g(t), h(t) \in P(F)$ suppose that $f(t)g(t) = h(t)$ and $h(t) = a(t - \lambda_1)^{m_1}(t - \lambda_2)^{m_2}...(t - \lambda_k)^{m_k}$, where $a, \lambda_1, \lambda_2, ..., \lambda_k \in F$ and each $m_i$ is a positive integer. Then show that $f(t) = b(t - \lambda_1)^{n_1}(t - \lambda_2)^{n_2}...(t - \lambda_k)^{n_k}$ for some $b \in F$ and for non-negative integers $n_i$ satisfying $n_i \leq m_i$. (Hint: Part (a) will be useful here.)

(c) For $f(t) \in P(F)$ if $f(t) = a(t - \lambda_1)^{m_1}(t - \lambda_2)^{m_2}...(t - \lambda_k)^{m_k}$ and $f(t) = b(t - \lambda_1)^{n_1}(t - \lambda_2)^{n_2}...(t - \lambda_k)^{n_k}$ for $a, b \in F$ and for non-negative integers $m_i, n_i$, then show that $a = b$ and $m_i = n_i$ for all $i=1,2,3,...,k$.

(2) Do the following problems at the end of section 7.1: #1,2(a)(b)(d),3(c),4.5.