MAT 310: HW13

(1) Let $P(F)$ denote the vector space of all polynomials in the one variable t with coefficients in the field F.

(a) For $f(t), g(t), h(t) \in P(F)$, if $f(t)h(t) = g(t)h(t)$ then show that $f(t) = g(t)$.

(b) For $f(t), g(t), h(t) \in P(F)$ suppose that $f(t)g(t) = h(t)$ and $h(t) = a(t-\lambda_1)^{m_1}(t-\lambda_2)^{m_2}....(t-\lambda_k)^{m_k}$, where $a, \lambda_1, \lambda_2, ..., \lambda_k \in F$ and each m_i is a positive integer. Then show that $f(t) = b(t-\lambda_1)^{n_1}(t-\lambda_2)^{n_2}.....(t-\lambda_k)^{n_k}$ for some $b \in F$ and for non-negative integers n_i satisfying $n_i \leq m_i$. (**Hint:** Part (a) will be usefull here.)

(c) For $f(t) \in P(F)$ if $f(t) = a(t-\lambda_1)^{m_1}(t-\lambda_2)^{m_2}....(t-\lambda_k)^{m_k}$ and $f(t) = b(t-\lambda_1)^{n_1}(t-\lambda_2)^{n_2}....(t-\lambda_k)^{n_k}$ for $a, b \in F$ and for non-negative integers m_i, n_i, then show that $a = b$ and $m_i = n_i$ for all i=1,2,3,...,k.

(2) Do the following problems at the end of section 7.1: #1,2(a)(b)(d),3(c),4.5.