**Instructions:** Fill in the blank lines on this sheet. Do any 4 of the following 5 problems in the spaces provided; do not do all 5 problems. Be sure to show some work, or give an explanation for each of your answers.

Print Name:

Write ID number:
(1) Set $f(x, y, z) = y^{-3} \cos^2(xz)$.
(a) What is the domain of $f$?
   \textbf{Solution:} \text{Domain}(f) = \{(x, y, z) : x, y, z \in \mathbb{R}, y \neq 0\}.

(b) Compute the partial derivatives $f_x(x, y, z), f_{yy}(x, y, z)$.
   \textbf{Solution:}
   
   \begin{align*}
   f_x(x, y, z) &= -2zy^{-3}\cos(xz)\sin(xz) \\
   f_{yy} &= 12y^{-5}\cos^2(xz)
   \end{align*}
(2) Let \( g(x, y) \) denote a function of 2 variables which satisfies

\[
g(1, 1) = -4
\]

\[
\nabla g(1, 1) = -2\mathbf{i} + \mathbf{j}.
\]

Consider the surface in 3 space defined by \( g(x, y) + z^2 = 0 \). Find the equation for the tangent plane to this surface at the point \((1,1,2)\) on the surface.

**Solution:** First note that since \( \nabla g(1, 1) = g_x(1, 1)\mathbf{i} + g_y(1, 1)\mathbf{j} \) it follows that

(i) \( g_x(1, 1) = -2, g_y(1, 1) = 1 \).

Set \( f(x, y, z) = g(x, y) + z^2 \); then

(ii) \( f_x(x, y, z) = g_x(x, y), f_y(x, y, z) = g_y(x, y), f_z(x, y, z) = 2z \).

Moreover the equation for the given surface can be written as \( f(x, y, z) = 0 \). Thus the equation for the desired tangent plane to this surface at \((1,1,2)\) is

(iii) \( f_x(1,1,2)(x-1) + f_y(1,1,2)(y-1) + f_z(x,y,z)(z-2) = 0 \).

Now, combining (i)-(iii) above we see that the equation for the tangent plane is

\[-2(x-1) + (y-1) + 4(z-2).\]
(3) Let \( r(t) \) denote the position of a moving particle in 3-dimensional space at time \( t \) which passes thru the point \( i - j - 2k \) at time \( t = 1 \). Suppose that the acceleration and velocity of the particle — denoted by \( a(t), v(t) \) — satisfy

\[
\begin{align*}
  a(t) &= -16 \cos(\pi t) j \\
  v(1) &= -2j - k.
\end{align*}
\]

Find the position of the particle at time \( t = 2 \).

**Solution:** \( v(t) \) is an antiderivative for \( a(t) = -16 \cos(\pi t) j \); thus

\[
(i) \quad v(t) = -\frac{16}{\pi} \sin(\pi t) j + u
\]

for some vector \( u \). Using this formula for \( t = 1 \) we get that

\[
(ii) \quad v(1) = u;
\]

so by combing (ii) with the hypothesis of this problem (that \( v(1) = -2j - k \)) we get

\[
(iii) \quad u = -2j - k.
\]

Now (i) and (iii) imply that

\[
(iv) \quad v(t) = (-\frac{16}{\pi} \sin(\pi t) - 2)j - k.
\]

Note also that \( r(t) \) is an antiderivative for \( v(t) \); thus by (iv) we get that

\[
(v) \quad r(t) = (\frac{16}{\pi^2} \cos(\pi t) - 2t)j - tk + w
\]

for some vector \( w \). We can solve for \( w \) by using (v) to compute \( r(1) \) and comparing it to the hypothesis for this problem (that \( r(1) = i - j - 2k \)). Thus we get that

\[
(vi) \quad w = i + (\frac{16}{\pi^2} + 1)j - k.
\]

Now combining (v) and (vi) we get that

\[
(vii) \quad r(t) = i + (\frac{16}{\pi^2} \cos(\pi t) - 2t + \frac{16}{\pi^2} + 1)j - (t + 1)k.
\]

To complete this problem use (vii) to compute \( r(2) \).
(4) Set \( f(x, y) = x \sin(y) \).

(a) Find all critical points for \( f(x, y) \).

**Solution:** Since \( f_x(x, y) = \sin(y) \) and \( f_y(x, y) = x \cos(y) \), the equations which define the critical points for \( f \) are

\[
\begin{align*}
\sin(y) &= 0 \\
x \cos(y) &= 0.
\end{align*}
\]

The solutions to these two equations are

\[(x, y) = (0, n\pi),\]

where \( n \) is equal to any integer.

(b) Use the second partial derivative test to determine the nature of each critical point for \( f(x, y) \).

**Solution:** \( f_{xx}(x, y) = 0, f_{xy}(x, y) = \cos(y), f_{yy}(x, y) = -x \sin(y) \).

Thus

\[d = f_{xx}(0, n\pi)f_{yy}(0, n\pi) - (f_{xy}(0, n\pi))^2 = -(\cos(n\pi))^2 = -1.\]

So each of the critical points \((0, n\pi)\) is a saddle point.
(5) Let $f(x, y, z)$ denote a function of 3-variables which satisfies
\[ \nabla f(0, 1, 2) = i - j + 3k. \]
Compute the directional derivative $D_u f(0, 1, 2)$ where $u$ is a unit vector defined by
\[ u = \sin(1)j - \cos(1)k. \]
(Express your answer in terms of $\sin(1), \cos(1)$.)
**Solution:** Use the formula
\[ D_u f(0, 1, 2) = \nabla f(0, 1, 2) \cdot u, \]
where $\cdot$ denotes dot product.