SOLUTIONS TO MIDTERM I; MAT 131 (SPRING, 08)

Instructions: Please complete each of the following 5 problems on the sheet of paper on which it appears. Answers given without any work shown or any reasons given will receive 0 credit.

(1) Set \( f(x) = \sqrt{x-2} \) and \( g(x) = \sqrt{2-x} \). Find a formula for each of the following functions and find its domain.

(a) \( f \circ g \).

Solution: \( f \circ g(x) = \sqrt{2-x-2} \); for \( x \) to be in the domain of \( f \circ g \) we need both \( 2-x \geq 0 \) and \( 2-x \geq 4 \). Thus \( \text{domain}(f \circ g) = (-\infty, -2] \).

(b) \( g \circ g \).

Solution: \( g \circ g(x) = \sqrt{2-\sqrt{2-x}} \); for \( x \) to be in the domain of \( g \circ g \) we need both \( 2-x \geq 0 \) and \( 4 \geq 2-x \). Thus \( \text{domain}(g \circ g) = [-2, 2] \).

(2) Set \( f(x) = 2x^2 + 6x + 4 \).

(a) Sketch the graph of \( f(x) \), indicating its intersection with the y-axis and the coordinates of the lowest point on the graph.

Solution: Note that \( 2x^2 + 6x + 4 = 2(x^2 + 3x + 2) = 2((x + \frac{3}{2})^2 - \frac{1}{4}) \). Thus the graph is an upward shaped bowl with its lowest point equal to \((-\frac{3}{2}, -\frac{1}{2})\). The y-intercept of this graph is \((0, 4)\).

(b) Where is the function increasing? And where is the function decreasing.

Solution: The function is increasing on \([-\frac{3}{2}, \infty)\) and is decreasing on \((-\infty, \frac{3}{2}]\).

(3) Compute the following limits.

(a) \( \lim_{x \to 2} \frac{x^2-2x}{x^2-x-2} \).

Solution: \( \frac{x^2-2x}{x^2-x-2} = \frac{x}{x+1} \) if \( x \neq 2 \). Thus the limit is equal to \( \lim_{x \to 2} \frac{x}{x+1} = \left( \lim_{x \to 2} \frac{x}{x+1} \right) \left( \lim_{x \to 2} \frac{x}{x+1} \right) = \frac{2}{3} \).

(b) \( \lim_{x \to -\infty} \frac{x^2-2x}{x^2-x-2} \).

Solution: \( \frac{x^2-2x}{x^2-x-2} = \frac{1-2/x}{1-1/x-2/x^2} \). Thus the limit is equal to \( \lim_{x \to -\infty} \frac{1-2/x}{1-1/x-2/x^2} = \frac{\lim_{x \to -\infty} 1-2/x}{\lim_{x \to -\infty} 1-1/x-2/x^2} = 1 \).
(4) Compute the tangent line to the graph of \( f(x) = \frac{x+1}{x-2} \) at the point (3,2).

**Solution:** The slope of this tangent line is equal to \( \lim_{x \to 3} \frac{x+1}{x-2} \).

\[
\lim_{x \to 3} \frac{x+1}{x-2} = \lim_{x \to 3} \frac{(-1)(x-3)}{(x-2)(x-3)} = \lim_{x \to 3} \frac{-1}{x-2} = \frac{-1}{1} = -1.
\]

Thus the equation for the tangent line at (3,2) is \( y - 2 = -1 \cdot (x - 3) \).

(5) A ball is thrown straight up. Its height above the ground \( t \) seconds later is \( 40t - 16t^2 \) feet.

(a) Find the (vertical) velocity of the ball when \( t=1 \) seconds. Is the ball moving up or down at \( t=1 \)?

**Solution:** Let \( v(t) \) denote the velocity at time \( t \). Then \( v(1) = \lim_{t \to 1} \frac{40t - 16t^2 - 24}{t-1} = \lim_{t \to 1} \frac{8(t-1)(-2t+3)}{t-1} = \lim_{t \to 1} 8(-2t + 3) = 8(-2 + 3) = 8. \) Since \( v(1) > 0 \) the ball is traveling up at \( t=1 \).

(b) Find the (vertical) velocity of the ball when \( t=2 \) seconds. Is the ball moving up or down at \( t=2 \)?

**Solution:** \( v(2) = \lim_{t \to 2} \frac{40t - 16t^2 - 16}{t-2} = \lim_{t \to 2} \frac{8(t-2)(-2t+1)}{t-2} = \lim_{t \to 2} 8(-2t + 1) = 8(-4 + 1) = -24. \) Since \( v(2) < 0 \) the ball is traveling down at \( t=2 \).