SOLUTIONS TO MIDTERM I; MAT 131 (SPRING, 08)

Instructions: Please complete each of the following 5 problems on the sheet of paper on which it appears. Answers given without any work shown or any reasons given will receive 0 credit.

(1) Set $f(x) = \sqrt{x-2}$ and $g(x) = \sqrt{2-x}$. Find a formula for each of the following functions and find its domain.

(a) $f \circ g$.

Solution: $f \circ g(x) = \sqrt{\sqrt{2-x} - 2}$; for x to be in the domain of $f \circ g$ we need both $2-x \geq 0$ and $2-x \geq 4$. Thus domain($f \circ g$)=$(-\infty, -2]$.

(b) $g \circ g$.

Solution: $g \circ g(x) = \sqrt{2 - \sqrt{2-x}}$; for x to be in the domain of $g \circ g$ we need both $2-x \geq 0$ and $4 \geq 2-x$. Thus $domain(g \circ g) = [-2, 2]$.

(2) Set $f(x) = 2x^2 + 6x + 4$.

(a) Sketch the graph of $f(x)$, indicating its itersection with the y-axis and the coordinates of the lowest point on the graph.
Solution: Note that $2x^2 + 6x + 4 = 2(x^2 + 3x + 2) = 2((x + \frac{3}{2})^2 - \frac{1}{4})$. Thus the graph is an upward shaped bowl with its lowest point equal to $(-\frac{3}{2}, -\frac{1}{2}))$. The y-intercept of this graph is $(0, 4)$.

(b) Where is the function increasing? And where is the function decreaing.
Solution: The function is increasing on $[-\frac{3}{2}, \infty)$ and is decreasing on $(-\infty, \frac{3}{2}]$.

(3) Compute the following limits.

(a) $limit_{x \to 2} \quad \frac{x^2-2x}{x^2-x-2}$.
Solution: $\frac{x^2-2x}{x^2-x-2} = \frac{x}{x+1}$ if $x \neq 2$. Thus the limit is equal to $lim_{x \to 2} \quad \frac{x}{x+1} = \frac{lim_{x \to 2} \quad x}{lim_{x \to 2} \quad x+1} = \frac{2}{3}$.

(b) $limit_{x \to -\infty} \quad \frac{x^2-2x}{x^2-x-2}$.
Solution: $\frac{x^2-2x}{x^2-x-2} = \frac{1-2/x}{1-1/x-2/x^2}$. Thus the limit is equal to $lim_{x \to \infty} \frac{1-2/x}{1-1/x-2/x^2} = \frac{lim_{x \to -\infty} 1-2/x}{\lim_{x \to -\infty} 1-1/x-2/x^2} = \frac{1}{1}$.

(4) Compute the tangent line to the graph of $f(x) = \frac{x-1}{x-2}$ at the point (3,2).

Solution: The slope of this tangent line is equal to $limit_{x\to 3}\frac{\frac{x-1}{x-2}-2}{x-3} = limit_{x\to 3}\frac{\frac{-x+3}{x-2}}{x-3} = limit_{x\to 3}\frac{(-1)(x-3)}{(x-2)(x-3)} = limit_{x\to 3}\frac{-1}{x-2} = \frac{limit_{x\to 3} \quad -1}{limit_{x\to 3} \quad (x-2)} = \frac{-1}{1} = -1$. Thus the equation for the tangent line at (3,2) is $\frac{y-2}{x-3} = -1$.

(5) A ball is thrown straight up. Its height above the ground t seconds later is $40t - 16t^2$ feet.

(a) Find the (verical) velocity of the ball when t=1 seconds. Is the ball moving up or down at t=1?
Solution: Let $v(t)$ denote the velocity at time t. Then $v(1) = limit_{t\to 1}\frac{40t-16t^2-24}{t-1} = limit_{t\to 1}\frac{8(t-1)(-2t+3)}{t-1} = limit_{t\to 1} \quad 8(-2t+3) = 8(-2+3) = 8$. Since $v(1) > 0$ the ball is traveling up at t=1.

(b) Find the (vertical) velocity of the ball when t=2 seconds. Is the ball moving up or down at t=2?
Solution: $v(2) = limit_{t\to 2}\frac{40t-16t^2-16}{t-2} = limit_{t\to 2}\frac{8(t-2)(-2t+1)}{t-2} = limit_{t\to 2}8(-2t+1) = 8(-4+1) = -24$. Since $v(2) < 0$ the ball is traveling down at t=2.

.