Some Notes from January 30

In class we were determining the fourth roots of -8.

We wrote, in exponential form, $-8 = 8e^{i\pi}$.

We obtain one fourth root by taking the (real, positive) fourth root of 8 and dividing the argument, π, by four. This is $\sqrt[4]{8}e^{i(\pi/4)}$.

Since the principal argument of -8 is π, this fourth root is called the principal fourth root. This is denoted c_0 in Churchill and Brown.

To find the other roots, consider the representation $-8 = 8e^{i\pi+i2k\pi}$. So each root is of the form $\sqrt[4]{8}e^{i(\pi+2k\pi)/4)}$, $k = 0, 1, 2, 3$.

Using notation consistent with your textbook, we have:

$$c_0 = \sqrt[4]{8}e^{i(\pi/4)}.$$
$$c_1 = \sqrt[4]{8}e^{i(3\pi/4)} = \sqrt[4]{8}e^{\pi i/4}e^{2\pi i/4} = c_0\omega_4.$$
$$c_2 = \sqrt[4]{8}e^{i(5\pi/4)} = \sqrt[4]{8}e^{\pi i/4}e^{4\pi i/4} = c_0\omega_4^2.$$
$$c_3 = \sqrt[4]{8}e^{i(7\pi/4)} = \sqrt[4]{8}e^{\pi i/4}e^{6\pi i/4} = c_0\omega_4^3.$$

Where $\omega_4 = e^{(2\pi i/4)}$, the principal fourth root of one.

- You should sketch a graph of these four roots.
- This problem gives us the function $f(z) = z^{1/4}$ as a first example of a *multi-valued* function; we will study other multi-valued functions in this course, so this is an important example to understand.
- For a practice problem that you could verify using only high school math, try finding the three third roots of 8. So you are solving $x^3 - 8 = 0$.