

MIDTERM 2 PART B

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit. Your exam will be graded based on the mathematics in your response and also on how well you communicate your mathematical ideas.

You are free to talk with any student in class about the questions on this part of the exam, but the solutions you submit must be your own writing and communicate your own understanding. You should not get help from any tutors from the MLC or elsewhere; you should not talk with anyone outside of class or consult resources other than the course textbook and your colleagues in this class.

This part of the midterm will be scaled to 15% of midterm 2.

- (1) Carefully read section 59 in the Churchill and Brown book.
- (2) The section starts with a lemma, stated on page 173. Compare this lemma to Liouville's Theorem, stated on the preceding page. Explain how these results are similar and how they are different. Focus on both the hypotheses and on the conclusions of the two results. Feel free to include any examples or non-examples to help communicate your ideas.
- (3) In the proof of this lemma, on page 174, your authors write: "We note from [equation] (2) that when a function is analytic within and on a given circle, its value at the center is the arithmetic mean of its values on the circle. This result is called *Gauss's mean value theorem*."

Carefully explain the authors' point in this statement.

- (4) Set $f(z) = e^{z^2}$, and determine the points at which the function $g(z) = |f(z)|$ obtains its maximum and minimum values on the closed region given by $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Explain completely.