

**MATH 301/501 HOMEWORK 2—DUE AT THE BEGINNING OF CLASS ON
TUESDAY, SEPTEMBER 21.**

There are two pages to this document.

- (1) For each statement, either prove the statement or provide a counter-example.
 - (a) The rational numbers are closed under addition.
 - (b) The irrational numbers are closed under addition.
 - (c) The rational numbers are closed under multiplication.
 - (d) The irrational numbers are closed under multiplication.

- (2) You ask your high school students to write a clear, concise definition of a function. Several student definitions are listed below. For each item, write a clear, thoughtful response to the student that communicates your assessment of the response.
 - (a) A function is an equation so that for each x there is only one y .
 - (b) A function is a graph that passes the vertical line test.
 - (c) A function is a mapping between sets, so that all of the x values map to only one y value.

- (3)
 - (a) Give an example of a function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ that is injective but not surjective. *Prove* your result.
 - (b) Give an example of a function $g : \mathbb{Z} \rightarrow \mathbb{Z}$ that is surjective but not injective. *Prove* your result.
 - (c) Give an example of a bijective function $h : \mathbb{Z} \rightarrow \mathbb{Z}$. *Prove* your result.
 - (d) Give an example of a function $s : \mathbb{Z} \rightarrow \mathbb{Z}$ that is neither injective nor surjective. *Prove* your result.

- (4) A student in your class is solving the equation
$$x^2 = 25.$$
The student concludes that $x = 5$.
 - (a) Clearly explain why the student is not correct.
 - (b) How could you help the student find the error and come to a correct solution.
 - (c) What is $\sqrt{25}$?

- (5) More may be posted by 7pm on Wednesday.