

MATH 501 FALL 2016
FINAL EXAM PART A

Name _____

You are allowed to work with other MAT 501 students on these problems. You may **not** discuss these problems with anyone else outside of class. In particular, you may not use the MLC tutors. The final write-up should be your own work and communicate your own understanding.

- (1) Let m , n , and ℓ denote three distinct parallel lines. Give a *precise* description of the composition of reflections: $r_m \circ r_n \circ r_\ell$. Prove your result. Make sure your proof will work for any configuration of three parallel lines.
- (2) Prove that any reflection is an isometry. That is, prove that, for any two points A and B , the distance $d(A, B)$ between point A and point B is equal to the distance $d(r_\ell(A), r_\ell(B))$, where r_ℓ denotes the reflection of the plane through line ℓ .
- (3) Prove that, by repeatedly composing only two reflections, one can generate each of the eight symmetries of a square. Can you do the same for an arbitrary n -gon, which has $2n$ symmetries?