(1) Give an example of a non-constant rational function $f(x)$ that satisfies the following:

(a) \[ \lim_{x \to \infty} f(x) = 0. \]

(b) There exists an $a$ in the domain of $f(x)$ that satisfies $f(a) = 0$.
(c) Sketch a graph of your function, on clearly labeled, scaled coordinate axes.

(2) Let $b$ denote a real number, $f(x)$ a real-valued function.

(a) Write a clear, concise definition of \[ \lim_{x \to b} f(x), \] that would be appropriate for a high school student.
(b) Write a definition that would be appropriate for an undergraduate student taking an analysis course.
(c) Explain precisely how these definition coincide.
(d) What does it mean for $f(x)$ to be a real-valued function?

(3) (a) Clearly state the domain and the range of the function $g(x) = \sin x$.
(b) Does $g(x)$ surject onto the real numbers?
(c) Is $g(x)$ an injective function?
(d) Clearly state the domain and the range of the function $f(x) = \sin^{-1} x$.
(e) Is it correct to write that $f(x)$ is the inverse of $g(x)$? Explain.

(4) A student writes: $\sin 60 = \sin (30 + 30) = \sin 30 + \sin 30 = \frac{1}{2} + \frac{1}{2} = 1$.

(a) Explain to the student, using the unit circle, that this is not correct.
(b) Explain to the student, using the graph of the function $g(x) = \sin x$, that this is not correct.
(c) Give the student an example of a familiar function $k(x)$ for which $k(A + B) = k(A) + k(B)$. Give the student a familiar example of a familiar function $h(x)$ for which $h(A + B) \neq h(A) + h(B)$. Explain why you think your choice of examples will help the student.