MATH 301/501 HOMEWORK 7–DUE AT THE BEGINNING OF CLASS ON TUESDAY, DECEMBER 1

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. A solution with little or no justification will receive little or no credit.

(1) Read through the two articles on visualizing complex zeros of quadratics.

N. Murray, *Making Imaginary Roots Real.*
J. Pastore and A. Sultan, *A Geometric Interpretation of Complex Zeros of Quadratic Functions.*

(2) In class we looked at the zeros of the quadratic polynomial $x^2 + 2x + c$, varying the parameter c. Try something similar with the quadratic $x^2 + bx + 12$. Vary the value of b and try to figure out how the zeros change–geometrically–as b varies. Look at a few examples with complex roots, as well as those with real roots. Explain your conclusions.

(3) One of the Standards for Mathematical Practice asks students to look for and make use of structure.

- Factor the polynomial $x^6 - 1$ by first treating it as a difference of perfect squares.
- Factor $x^6 - 1$ by first treating it as a difference of cubes.
- Sketch the graph of $f(x) = x^6 - 1$ in the real plane.
- Factor $x^6 - 1$ completely over the real numbers and over the complex numbers.
- Write all six of the complex roots in rectangular coordinates and in exponential form.
- Graph the roots in the complex plane.

(4) Find three complex numbers, z_1, z_2, and z_3, on the unit circle, for which the sum $z_1 + z_2 + z_3 = 0$, and graph them in the complex plane. Prove that the vertices form an equilateral triangle. Find a different triple w_1, w_2, w_3 that satisfies the same property, and sketch these points on a different graph. Prove that the vertices form an equilateral triangle.

Are there other triples of complex numbers, on the unit circle, that sum to zero? Will the points always form an equilateral triangle? Prove your result.