MATH 301/501 HOMEWORK 4–DUE AT THE BEGINNING OF CLASS ON TUESDAY, OCTOBER 6

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. A solution with little or no justification will receive little or no credit. **This document has 3 pages**.

(1) (a) Read the teacher version of Lesson 18 from Module 3 of the New York State Precalculus and Advanced Topics Modules, linked below.

(b) On page 298 of the module the authors graph the functions $y = t(v)$ and $y = v(t)$. Keeping in mind the ideas from the article on inverse functions, describe mathematical problems with this graph.

(c) Comment on the meaning of the intersection point, as discussed in the module.

(d) Find and carefully explain 1-2 other mathematical inconsistencies or misconceptions that are embedded in this module.

`https://www.engageny.org/resource/precalculus-and-advanced-topics-module-3-topic-c-lesson-18`.

(2) Write an equation that defines a non-constant rational function f that satisfies the following:

(a)
$$\lim_{x\to\infty} f(x) = 0.$$

(b) There exists an a in the domain of f for which $f(a) = 0$.

(c) Sketch a graph of your function, on clearly labeled, scaled coordinate axes.

(3) Write an equation for a rational function whose graph has three disjoint components, and sketch the graph on clearly labeled, scaled coordinate axes.

(4) (a) Write an equation for a rational function g, that is not a polynomial, that satisfies all of the following:
- $\lim_{x\to-\infty} g(x) = -\infty$.
- $\lim_{x\to\infty} g(x) = \infty$.
- The function g has no asymptotes.

(b) Sketch a graph of g on clearly labeled and scaled coordinate axes.

(5) Write a rational function $g(x)$ to fit the last graph we were looking at in class. It had two vertical asymptotes, say $x = a$ and $x = -a$. The graph passes through the origin, and this is the only zero of g. The function g approaches infinity as x approaches a from the right and from the left; g approaches $-\infty$ as x approaches $-a$ from the left and from the right. The line $y = 0$ is the unique horizontal asymptote. Finally, as x approaches infinity, $g(x)$ approaches zero from above, and as x approaches $-\infty$, $g(x)$ approaches zero from below.

(6) (a) Look at the Regents exam problems and solutions below

(b) Think carefully about **mathematical** inconsistencies among the problems and their proposed solutions. You don't need to turn in a written solution, but please come to class on Tuesday with your ideas to discuss.

New York State Regents Exam Problems

(a) The expression

$$\frac{\frac{x}{x+2}}{1-\frac{x}{x+2}}$$

is equivalent to:

(i) $\frac{2}{x}$

(ii) $\frac{x}{2}$

(iii) $\frac{2x}{x+2}$

(iv) $\frac{2x}{x^2+4}$.

Solution is given as (ii).

(b) When simplified, the complex fraction

$$\frac{1+\frac{1}{x}}{\frac{1}{x}-x},$$

$x \neq 0$, is equivalent to:

(i) 1

(ii) -1

(iii) $\frac{1}{1-x}$

(iv) $\frac{1}{x-1}$

Solution is given as (iii).

(c) For all values of x for which the expression is defined,

$$\frac{2x+x^2}{x^2+5x+6}$$

is equivalent to:

(i) $\frac{1}{x+3}$

(ii) $\frac{x}{x+3}$

(iii) $\frac{1}{x+2}$

(iv) $\frac{x}{x+2}$

Solution is given as (ii).