MAE 301/501 HOMEWORK–4 DUE AT THE BEGINNING OF CLASS ON THURSDAY, OCTOBER 10

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

This document has 2 pages.

(1) (a) Find several lessons from the NY State modules, (there are at least four), that directly address simplifying rational expressions/rational functions and solving equations involving rational expressions/rational functions.

(b) For each lesson, look carefully at how the modules treat rational expressions. Are they treated just as algebraic objects to manipulate, or are they also viewed as real-valued functions. Is the distinction treated consistently throughout?

(c) Write several paragraphs explaining your findings. The paragraphs should clearly communicate your mathematical understanding of rational expressions and rational functions.

(2) For the problems below, you can choose to use the examples from class or to come up with new ones.

(a) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ that is injective, but not surjective, and prove your result.
(b) Give an example of a function $g : \mathbb{Z} \to \mathbb{Z}$ that is surjective, but not injective, and prove your result.
(c) Give an example of a function $h : \mathbb{Z} \to \mathbb{Z}$ that is injective and surjective, and prove your result.
(d) Give an example of a function $k : \mathbb{Z} \to \mathbb{Z}$ that is neither injective nor surjective, and prove your result.

(3) Let f denote a rational function. The line defined by an equation of the form $y = mx + b$ is a *slant asymptote* of the graph of f if

$$\lim_{x \to \infty} (f(x) - (mx + b)) = 0$$

or if

$$\lim_{x \to -\infty} (f(x) - (mx + b)) = 0$$

Determine whether or not the graph of a rational function can cross its slant asymptote. Either *prove* that this is not possible or give an *explicit* example, along with a *clear* sketch of the graph, to show that it is possible. In either case, carefully *explain* your solution and process.

(4) In class students gave an example of a function with a vertical asymptote at $x = 0$, for which $f(0) = 0$.

Think about whether you can construct a rational function g that satisfies both of the following.

(a) The line $x = a$ is a vertical asymptote.

(b) There is some b in the domain of g, for which $g(b) = a$.

Either give an explicit example or prove that this is not possible.