

**MATH 301/501 HOMEWORK 3—DUE AT THE BEGINNING OF CLASS ON
THURSDAY, SEPTEMBER 24**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. A solution with little or no justification will receive little or no credit.

- (1) Please do this part for Tuesday: Read the article "Inverse Functions: What Our Teachers Didn't Tell Us," by Wilson, Adamson, Cox and O'Bryan. This is published in *Mathematics Teacher*, March 11, Vol. 104, Issue 7. If you can't find the article, here is a link to a related article by the same authors:

<https://blogs.ams.org/matheducation/2016/11/28/inverse-functions-were-teaching-it-all-wrong/>

- (2) (a) Write a clear, concise definition of an injective function.
(b) Write a clear, concise definition of a surjective function.
- (3) (a) Give an example of a function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ that is injective, but not surjective, and prove your result.
(b) Give an example of a function $g : \mathbb{Z} \rightarrow \mathbb{Z}$ that is surjective, but not injective, and prove your result.
(c) Give an example of a function $h : \mathbb{Z} \rightarrow \mathbb{Z}$ that is bijective, and prove your result.
(d) Give an example of a function $k : \mathbb{Z} \rightarrow \mathbb{Z}$ that is neither injective nor surjective, and prove your result.
- (4) (a) Suppose $f(x)$ and $g(x)$ are real-valued functions with domain consisting of the real numbers. Suppose f and g are both injective functions. Determine whether or not $g(f(x))$ must also be injective, and prove your result.
(b) Suppose $f(x)$ and $g(x)$ are real-valued functions with domain consisting of the real numbers. Suppose f and g both surject onto the real numbers. Determine whether or not $g(f(x))$ must also be surjective, and prove your result.
(c) Give an example of two functions, $f(x)$ and $g(x)$, coming from the high school mathematics curriculum, for which $g(x)$ is injective but $g(f(x))$ is not injective. Explain.
- (5) (a) How many radians in $\pi/3$ degrees?
(b) How many degrees in 60 radians?
- (6) For each statement below, determine whether the equality is correct, and explain your solution.
(a) $\sin(\sin^{-1} x) = x$
(b) $\sin^{-1}(\sin x) = x$