MAE 301/501 HOMEWORK–2 DUE AT THE BEGINNING OF CLASS ON THURSDAY, SEPTEMBER 12

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

(1) (a) For the polynomial $x^4 - 4$:

- Find all roots and graph them in the complex plane.
- Factor the polynomial completely over the rational numbers.
- Factor the polynomial completely over the real numbers.
- Factor the polynomial completely over the complex numbers.

   (b) Do the same for the polynomial $x^4 + 4$.

   (c) What are the principal fourth roots of 4 and of -4? Explain.

(2) In class we determined that $x^3 - 1 = (x-1)(x^2 + x + 1)$. Generalize this identity to the polynomial $x^n - 1$, and *prove* that your result holds for any n.

(3) (a) For a polynomial $x^2 + bx + c$, give the coefficients in terms of its roots: α_1 and α_2.

   (b) For a monic, cubic polynomial, give the coefficients in terms of its roots.

   (c) Generalize these result to monic polynomials of higher degree

(4) Let $\{\alpha_1, \alpha_2, \cdots \alpha_n\}$ denote the roots of a degree n polynomial. Define the discriminant of the polynomial as:
$$D := \prod_{i<j} (a_i - a_j)^2.$$

   (a) For a quadratic polynomial, explain how the value of the discriminant, defined here, tells you the nature of the roots of the quadratic.

   (b) For a quadratic polynomial, explain how this definition of discriminant relates to the discriminant as it is defined in high school algebra.

   (c) For a cubic polynomial, explain how the discriminant characterizes the nature of its roots.