# MAE 301/501 HOMEWORK–1 DUE AT THE BEGINNING OF CLASS ON THURSDAY, SEPTEMBER 3

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

(1) Given a right triangle, whose side lengths are relatively prime: $\gcd(a, b, c) = 1$. Prove that the hypotenuse is odd and that the two legs are of opposite parity: one odd and one even.

(2) A circle can be defined as the set of all points whose distance from a fixed point is a constant. Use this definition to derive the equation for a circle with center $(h, k)$ and radius $r$.

(3) For the second half of class on Thursday, we were finding the intersection of the line through the points $(0, 1)$ and $(t, 0)$ with a unit circle, centered at the origin. (I will post some notes about where we left off.)

  (a) Write the coordinate $x$ in terms of $t$.

  (b) Write the coordinate $y$ in terms of $t$.

  (c) We chose $t$ to be rational. Explain why $x$ and $y$ must also be rational.

  (d) Explain how we can use this method to construct infinitely many primitive Pythagorean triples.

(4) Let's construct a different proof that there are infinitely many Pythagorean triples.

  (a) Take $a$ to be the even leg and $b$ to be the odd leg. I can rewrite our equation as $(c-a)(c+a) = b^2$. Prove that the greatest common divisor of $(c - a)$ and $(c + a)$ is 1.

  (b) Using the fact that $(c - a)$ and $(c + a)$ are relatively prime, (have greatest common divisor of 1), prove that $(c-a)$ and $(c+a)$ are both perfect squares. That is, $c-a = r^2$, for some integer $r$, and $c + a = s^2$, for an integer $s$. Why is it necessary for the proof that $c - a$ and $c + a$ are relatively prime?

  (c) Now go backwards: explain how you can choose relatively prime integers $s$ and $r$ to generate infinitely many primitive Pythagorean triples.

(5) Give a clear proof of the Pythagorean theorem. For this problem, it is okay to consult outside sources. You should site any source you use. I am interested in how well you **explain** and understand the proof.