

**MAT 401 HOMEWORK-2 DUE *IN CLASS*, AT THE BEGINNING OF CLASS, ON  
THURSDAY, SEPTEMBER 26**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

- (1) Please make sure you have read through page 56 in the course book by September 19. In particular, pages 43 - 47 are not being covered via the student presentations; please make sure you read this on your own.
- (2) In class on September 12, Yuhao gave a computation of the genus of the hyperelliptic curves we have been looking at. This computation can be extended/generalized to prove the Riemann-Hurwitz Theorem:

**Theorem.** *Let  $f : X \rightarrow Y$  denote a degree  $d$  morphism of compact Riemann surfaces of genera  $g$  and  $g'$ , respectively. Then:*

$$2g - 2 = d(2g' - 2) + \sum_{x \in X} (m_x(f) - 1).$$

- (a) Go back to the exercise you did, to prove that the example in 1.33 is indeed an unramified morphism, and verify the Riemann-Hurwitz Theorem for this morphism.
  - (b) If you want, work out the details to example 1.62 or example 1.63.
- (3) Go through example 1.51, to make sure you understand the process of transforming a polygonal representation of a Riemann surface into its normalized form. **You do not need to submit this problem.**
  - (4) Connor briefly reviewed the definition of a normal subgroup and of a factor group. Let  $S_3$  denote the group of permutations of a set with 3 elements.
    - (a) What are the subgroups of this group?
    - (b) Which of the subgroups are normal subgroups?
    - (c) For each of the normal subgroups  $H$ , describe the factor group  $S_3/H$ .
  - (5) In example 1.66 of the book, the authors write that the truncated helix is not a covering of  $S^1$ . Carefully explain how the given map from the truncated helix to  $S^1$  fails to satisfy the *definition* of covering. (You should replace *topological surface* with *topological space*, in the definition.)
  - (6) **More to be posted by the end of the day on September 20.**