MAT 319 HOMEWORK–9 DUE AT THE BEGINNING OF CLASS ON WEDNESDAY, NOVEMBER 7

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit. Clear, organized, partially correct work may receive partial credit.

(1) Carefully (re)-read through section 19 from the textbook. If you have time, start reading section 20.

(2) (a) Re-read the proof of Theorem 18.5 from the textbook.

(b) In the proof, the authors write that the proof would need to be modified at the endpoints of the interval. Think about what would need to be modified, and explain your ideas.

(c) Your authors sketch a proof of Theorem 18.4. Fill in details for a complete proof, including a clear explanation of how Theorem 18.5 is used.

(3) (a) Give an example of a function f with domain $[0, 1]$ for which, given y in the image of f, $f^{-1}(y)$ consists of exactly two points.

(b) Determine whether such a function can be continuous, and prove your result.

(4) For each function below, determine whether it is uniformly continuous on the given interval, and prove your result.

(a) $f(x) = \frac{1}{x}$ on $[2, \infty]$.

(b) $g(x) = \frac{1}{x}$ on $(0, 1)$.

(5) Determine whether the sum of two uniformly continuous functions must be uniformly continuous, and prove your result.

(6) Do problem 19.4.

(7) Finish the problem we were working on in class: prove, using the definition of uniform continuity, that the function $f(x) = x^2$ is not uniformly continuous on $[0, \infty)$.