MAT 319 HOMEWORK–7 DUE AT THE BEGINNING OF CLASS ON WEDNESDAY, OCTOBER 24

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit. Clear, organized, partially correct work may receive partial credit.

This document has two pages.

(1) Carefully (re)-read sections 14 and 15 from the textbook.

(2) Give an example of a sequence, give an example of a series, and explain in a few sentences the difference between a sequence and a series.

(3) Sierpinska Triangle. Consider the equilateral triangle below, and suppose it has area exactly one.

  (a) Remove the "middle" white triangle. (This was formed by joining the midpoints of the sides of the original, larger, triangle), and write the area of what remains.

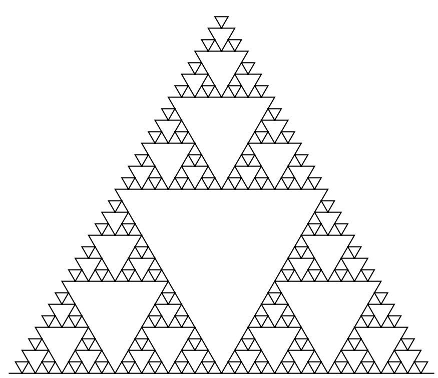
  (b) Next remove the middle white triangle from each of the three remaining pieces, and write the area of what remains.

  (c) Next remove the middle white triangle from each of the nine remaining pieces, and write the area of what remains.

  (d) Suppose you continue this process, with infinitely many iterations.

    (i) Express the total *removed* area as an infinite series.

    (ii) Find the area of what would remain, after "infinitely many" iterations. Explain



(4) Do any one of the following: (Do all if you have time!)

(a) Determine whether or not the series below is absolutely convergent and explain your result.

$$\sum_{n=1}^{\infty} \frac{(-2)^n n!}{(2n)!}$$

(b) Find the values of p for which the following series is convergent and explain your result.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

(c) Determine whether the series below converges or diverges, and explain your reasoning.

$$\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{3^n + 5^n}}$$

(5) Do any two of the following:

(a) Let $\sum a_n$ and $\sum b_n$ denote two series. Suppose that there is a number N for which, for all $n \geq N$, $a_n = b_n$. Prove that the series $\sum a_n$ converges iff the series $\sum b_n$ converges.

(b) Problem 14.14
(c) Problem 15.4
(d) Problem 15.6

(6) Suggested–Do probem 14.3