

MAT 312 HW 1

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

- (1) For each pair of integers, find the greatest common divisor d , and write d in the form $d = ar + bs$ for appropriate s and t .

- (a) $a = -28, b = -63$.
- (b) $a = 630, b = 132$.
- (c) $a = 7245, b = 4784$.

- (2) Find the greatest common divisor of 6, 14, and 21, and write it in the form $6r + 14s + 21t$, for appropriate r, s and t .

- (3) Find integers a, b , and c for which $a|bc$ but $a \nmid b$ and $a \nmid c$.
(The symbol \nmid means “does not divide.”)

- (4) Suppose that $(a, c) = 1$ and that $(b, c) = 1$. Use this to prove that $(ab, c) = 1$.

- (5) Suppose you have two jugs. One holds exactly 12 units of water, the other holds exactly 17 units. The jugs are not marked with intermediate measurements. Explain how to use these jugs to measure exactly 8 units of water.

- (6) Find a formula for the sum of the first n odd positive integers.

- (7) Suppose $x \neq 1$ and that n is a positive integer. Prove that

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}.$$

- (8) (a) Prove that, for every positive integer n , $5|(n^5 - n)$.

- (b) Prove that, for every positive integer n , $8|(3^{2n} - 1)$.