MAT 534: MIDTERM

TU, OCT 14, 2014

Your name:____

(please print)

	1	2	3	**Total**
Grade				

1. Classify all groups of order $5 \times 7 \times 11$

2. Let $X \subset \mathbb{Z}^n$ be a subgroup of free rank n.

(a) Prove that $\mathbb{Z}^n/X$ is finite.

(b) Let $X^\vee = \{\lambda \in \mathbb{R}^n \mid \lambda \cdot x \in \mathbb{Z} \text{ for any } x \in X\}$. Prove that then $X^\vee$ is a free abelian group of rank n, and $X^\vee/\mathbb{Z}^n$ is finite.

(c) (Optional – do only after you have completed the other problems) Prove that $\mathbb{Z}^n/X \simeq X^\vee/\mathbb{Z}^n$.

3. Let $\mathbb{F}$ be a field; denote $R = \{\frac{f}{g} \mid f, g \in \mathbb{F}[x]\}$ — the fraction field of $\mathbb{F}[x]$. Choose an element $a \in \mathbb{F}$ and set

$$R_a = \left\{\frac{f}{g} \in R \mid g(a) \neq 0\right\}.$$

(a) Show that the set I of all non-invertible elements $x \in R_a$ is a maximal ideal. Describe the quotient R_a/I.

(b) Show that R_a is a principal ideal domain and describe all ideals of R_a.