1. Which of the following relations on the set $\mathbb{R}$ are equivalence relations? For each relation that is an equivalence relation, describe the equivalence class $[\sqrt{2}]$.
   (a) $x \sim y$ if $xy \geq 0$
   (b) $x \sim y$ if $x + y = 1$
   (c) $x \sim y$ if $x - y \in \mathbb{Q}$ (here $\mathbb{Q}$ is the set of rational numbers)
   (d) $x \sim y$ if $x^2 = y^2$.
   (e) $x \sim y$ if $(x = y$ or $xy = 1)$.

2. Recall the equivalence relation on the set $\mathbb{Z}$ defined by
   \[ a \equiv b \mod n \quad \text{if} \quad n \mid a - b \]
   (it is called “congruence modulo $n$”).
   (a) Prove that if $a \equiv b \mod n$ and $c \equiv d \mod n$, then $a + c \equiv b + d \mod n$ (in other words, congruences can be added).
   (b) Prove that if $a \equiv b \mod n$ and $c \equiv d \mod n$, then $ac \equiv bd \mod n$ (in other words, congruences can be multiplied).
   (c) Show that it is possible that $ab \equiv 0 \mod n$, but $a \not\equiv 0 \mod n$, $b \not\equiv 0 \mod n$.

3. Let $A = \mathbb{R} \times \mathbb{R} - \{(0,0)\}$ be the plane with the origin removed. Consider the following relation on $A$:
   \[ (x_1, y_1) \sim (x_2, y_2) \quad \text{if} \quad \exists t \in \mathbb{R} > 0: x_2 = tx_1, y_2 = ty_1 \]
   (a) Prove that this is an equivalence relation.
   (b) Describe and plot on the coordinate plane the equivalence class $[(1,1)]$.

4. Let $A = \mathbb{Z} \times \mathbb{Z}$. Consider the following relation on $A$:
   \[ (x_1, y_1) \sim (x_2, y_2) \quad \text{if} \quad x_1 - x_2 \text{ is even, } y_1 - y_2 \text{ is even} \]
   (a) Prove that this is an equivalence relation.
   (b) Describe the equivalence class $[(1,1)]$.
   (c) How many equivalence classes are there?