MAT 360: HOMEWORK 9

DUE WED, NOV 18

These problems are about affine transformations.

Definition 1. An affine trasformation is a bijection T fo the plane to itself such taht for any line l, $T(l)$ is also a line.

We have proved the following results (T is an affine transformation):

1. If lines l, m are parallel, then $T(l), T(m)$ are also parallel
2. T sends parallelograms to parallelograms
3. If A, B, C, D are such that $AB \parallel CD$, and $A' = T(A)$, etc, then
$$\frac{A'B'}{C'D'} = \frac{AB}{CD}$$
4. For any two triangles $\triangle ABC$, $\triangle A'B'C'$, there exists an affine transformation T such that $T(A) = A'$, $T(B) = B'$, $T(C) = C'$.

As a corollary we see that for example T sends medians of triangles to medians.

Examples of affine transformations include

1. Isometries
2. Homotethies (dilations)
3. Shear transformation ($(x, y) \mapsto (x + cy, y)$), for some coordinate system on the plane.

Homework

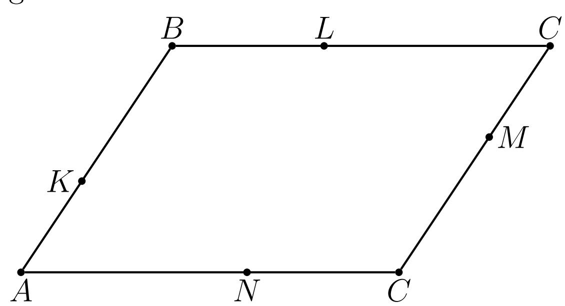
1. (a) Prove that A, B, C are not on the same line, and T is an affine transformation such that $T(A) = A$, $T(B) = B$, $T(C) = C$, then T is identity.

(b) Prove that for any two triangles $\triangle ABC$, $\triangle A'B'C'$, an affine transformation T satisfying $T(A) = A'$, $T(B) = B'$, $T(C) = C'$ is unique.

2. Prove that any affine transformation can we written as a composition of a isometries, dilations, and shear transformations (hint: use the previosu problem).

3. Consider a parallelogram $ABCD$ and mark points K, L, M, N on sides AB, BC, CD, DA dividing these sides in proportion $2 : 3$.

Prove that then $KLMN$ is also a parallelogram, and the intersection point of diagonals of $ABCD$ coincides with the intersection point of diagonals of $KLMN$.



[Hint: what if $ABCD$ is a square?]

4. Consider triangle $\triangle ABC$. Let point M be on side BC such that $BM : MC = 4 : 5$ and let K be on the continuation of side AB so that $BK : AB = 1 : 5$. Let N be the intersection point of lines KM and AC. Find $CN : AN$.