1. Prove the identity: \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \).

2. Consider the function \( f : \mathbb{R} \to \mathbb{R} \) given by \( f(x) = x^2 - x \)
   (a) Graph it.
   (b) Find \( f(A) \) where \( A \) is the open interval \((0, 4)\).
   (c) Find \( f^{-1}(B) \) where \( B = [1, 4] \).
   (d) Find two subsets \( C, D \) of \( \mathbb{R} \) such that \( f(C) \cap f(D) \neq f(C \cap D) \).

3. Let \( f : A \to B \) be a function and suppose that \( C \subseteq A \) and \( D \subseteq B \). Are the following statements true or false (for every choice of \( f, C, D \))? Justify your answers by a brief proof or a counterexample.
   (a) \( f(A \setminus C) \subseteq f(A) \setminus f(C) \).
   (b) \( f^{-1}(B \setminus D) = f^{-1}(B) \setminus f^{-1}(D) \).
   **Hint:** as in question 2, try some examples. You can try functions \( f : \mathbb{R} \to \mathbb{R} \) or you can try functions \( f : A \to B \) where \( A \) and \( B \) are finite sets.

4. Suppose that \( f : A \to B \) and \( g : B \to C \) are functions such that the composition \( g \circ f \) is surjective. Is \( g \) necessarily surjective? What about \( f \)? Give brief proofs or counterexamples.

5. Prove that for any positive integer \( n \),
   \[
   1^2 + 3^2 + \cdots + (2n-1)^2 = \frac{4n^3-n}{3}
   \]

6. Prove by induction that for any \( n \geq 5, n^2 < 2^n \). [Hint: prove first that \((n+1)^2 < 2n^2\).]

7. Guess a general formula for the product
   \[
   \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{n^2}\right)
   \]
   and prove it by induction.

8. Let the Fibonacci sequence be defined by \( F_1 = F_2 = 1, F_{n+1} = F_n + F_{n-1} \) for \( n > 1 \).
   Prove that then
   \[
   F_n F_{n+1} = F_1^2 + F_2^2 + \cdots + F_n^2
   \]