MAT 319: REVIEW SHEET FOR FINAL EXAM

Basic information

Time and place: Wed, 12/20, 8:00–10:30am, in our usual classroom.
Help session: Tu, 12/19, 3-4:30 pm, Math P–131

The final exam will be cumulative — it will cover all the material we have studied, up to derivateives and mean value theorem. It will be relatively short — no more than 7 questions.

The final will be **open book**. That is, you are allowed to use the textbook, your notes, previous exams with solutions and homeworks. You will not be allowed to use other books or computer resources.

The best way to prepare for the exam is to go over all the homeworks. On the next page, there are some additional practice problems.

Here is a brief listing of the main topics we have covered – nad which might appear in the final exam:

- Sets and functions. Countable and uncountable sets.
- Mathematical induction
- Axioms of real numbers. Completeness axiom; Archimedean principle. Supremum and infinum.
- Definition of limit of a sequence. Limit laws. Bolzano-Weierstrass Theorem. Subsequences. Cauchy criterion; contracting sequences. Properly divergent sequences.
- Series; covergence tests (including ratio test!).
- Limits of functions; cluster points of sets. One-sided limits and limits involving infinity.
- Continuous functions. Maximum/minimum of a continuous function on a closed interval; Intermediate Value Theorem.
- Derivatives. Definition; sum, product, and quotient rules. Chain rules; derivatives of rational and triginometric functions. Relation between derivative, local max/min, and increasing/decreasing functions. Mean Value Theorem.

1. Let $g_1\colon A \to B$, $g_2\colon A \to B$, $f\colon B \to C$ be functions such that $f \circ g_1 = f \circ g_2$.
(a) Prove that if f is injective, then $g_1 = g_2$.
(b) Show that if f is not injective, then it is possible that $g_1 \neq g_2$.

2. Use mathematical induction to prove that $n^3 + 3n^2 - n$ is divisible by 3 for all $n \geq 0$.

3. Let $f\colon A \to \mathbb{R}$ be a any function and let $s = \sup\{f(x) \mid x \in A\}$. Show that there exists a sequence $x_n \in A$ such that $\lim f(x_n) = s$.

4. (a) Let a_n be a sequence such that $a_n > 0$ and $\lim \frac{a_{n+1}}{a_n} = r > 1$. Prove that then sequence a_n is properly divergent: $\lim a_n = \infty$.
(b) Let a_n be a sequence such that $a_n > 0$ and for all n, $\frac{a_{n+1}}{a_n} > 1$. Does it necessarily mean that sequence a_n is properly divergent: $\lim a_n = \infty$?

5. Let $\sum a_n$ be a convergent series. Prove that then $\lim_{n\to\infty}\left(\sum_{i=n+1}^{2n} a_i\right) = 0$.

6. Prove **from definition** (i.e., without using limit laws) that

$$\lim_{x\to 4} \frac{x}{x-2} = 2$$

7. Let $f(x) = \sin(\pi x^2) - \sin(\pi x)$. Prove that there is a point $c \in \mathbb{R}$, $c > 0$ such that $f'(c) = 0$.