MAT 319: FINAL REVIEW

DEC 13, 2016

1. Properties of $\mathbb{R}$

- Basic properties of $\mathbb{R}$. Inequalities. Absolute value and Triangle inequality.
- Bounded sets. Notion of upper bound and lower bound.
- inf, sup. Completeness axiom.
- Archimedean property. Density of rational and irrational numbers in $\mathbb{R}$.

2. Limits of sequences 1

See handout 1 for details

- Definition of limit; convergent sequences. Computing limit from definition. Uniqueness of limits.
- Basic properties:
  Theorem: every convergent sequence is bounded.
  Theorem: if for all n, $a_n \geq A$, then $\lim a_n \geq A$ (if the limit exists).
- Comparison theorem for convergent sequences. Sum, product, and quotient rules
- Limits equal to $\pm\infty$.

3. Limits of sequences 2

The results below are based on the use of Completeness Axiom for real numbers.

- Every bounded monotone sequence converges.
- Cauchy sequences and Cauchy criterion: a sequence converges if and only if it is Cauchy.
- Nested intervals property.
- Subsequences; subsequential limits. Theorem: if $\lim a_n$ exists, then any subsequence has the same limit. A point is a subsequential limit of s_n if and only if in any neighbourhood of the point there are infinitely many terms of the sequence.
- Bolzano-Weierstrass theorem: any bounded sequence contains a convergent subsequence.

Note: notion of $\limsup$ and $\liminf$ will not be tested in the final.

4. Series

- Definition of a convergent series. Convergence of geometric series. Theorem: $\sum a_n$ converges $\implies \lim a_n = 0$
- Comparison test for series (proof will not be required). Ratio test. Root test (proof not required).
- Integral test. Convergence of series $\sum \frac{1}{n^p}$.
- Alternating series test
- Infinite decimals as series.

5. Continuous functions and limits of functions

- Definition of continuous function. Theorem: sum, product, quotient, composition of continuous functions are continuous. Examples: polynomials; exponential, logartithmic, trigonometric functions (no proof except for polynomials).
- Extreme value theorem.
- Intermediate value theorem.
- Limit of a function: definition using subsequences and ε-δ definition. One-sided limits. Continuity of piecewise defined functions.

6. Derivatives

- Definition of derivative. Differentiable $\implies$ continuous. Examples of differentiable/non-differentiable functions.
- Differentiation rules: sum rule, product rule, etc (understand proofs, be able to use them to prove similar statements from definition). Chain rule (statement only; proof is not required).
- Derivatives and maxima/minima of a function. Local max/min $\implies f' = 0$.
- Rolle theorem and mean value theorem.
- Increasing and decreasing functions. Detect increasing/decreasing via derivatives (know proofs, be able to prove related statements).

7. Integrals

- Definition of upper and lower Darboux sum. Behaviour of $L(f, P)$, $U(f, P)$ when we change (subdivide) a partition. Computing $U(f)$, $L(f)$ for simplest functions.
- Definition of integrable function. Theorem: any monotone bounded function is integrable. Theorem: any continuous bounded function is integrable (proof not required).