Homework 2, due on Thursday, September 15

1. Prove that the following statements about real numbers $a$, $b$, $c$ are equivalent to each other:

   (1) $a \in (b - c, b + c)$,
   (2) $b - c < a < b + c$,
   (3) $|a - b| < c$,
   (4) $b \in (a - c, a + c)$,
   (5) $c \in (\max(b - a, a - b), +\infty)$.

2. Let $A$ and $B$ be nonempty bounded subsets of $\mathbb{R}$, and let $A + B$ and $A - B$ be, respectively, the set of all sums $a + b$ and all differences $a - b$, where $a \in A$ and $b \in B$. Prove that $\sup(A + B) = \sup A + \sup B$ and $\sup(A - B) = \sup A - \inf B$.

3. Modify and prove the statements of the preceding problem for the products and quotients. Should any additional restrictions on the sets be imposed?

4. Problem 4.4 from the textbook.