MAT 200: HOMEWORK 5 (OPTIONAL)

This homework is optional. It contains practice problems for the final exam, covering all the material we had studied so far. It will not be graded.

In all the problems, unless stated otherwise, you can use any results contained in the book or given in class, but nothing more.

1. A (fictional) tax form contains the following statement:

> You can use the tax deduction if you do not itemize deductions and your adjusted gross income is less than \$54, 000 (\$61, 000 for married filing jointly).

Write this statement in the language of formal logic using notation
D: you can use the tax deduction
I: you itemize deductions
M: your filing status is "married filing jointly"
P: your adjusted gross income is less than \$54, 000
Q: your adjusted gross income is less than \$61, 000

2. Consider the following collection of statements about chess:

(a) A chess grandmaster can be defeated only by another grandmaster
(b) There lives at least one chess grandmaster in Philippines
(c) Bobby Fisher lives in Philippines
(d) Bobby Fisher can beat in chess any other person in Philippines

(A grandmaster is a title of a chess player; you do not need to know anything about chess or about chess titles to do this problem)

(a) Show that Bobby Fisher is a chess grandmaster, using only statements (a)–(d) and laws of logic. You do not have to write a formal proof; however, at each step please refer to the statement(s) you are using.
(b) Write statements (a)–(d) in the language of logic, using P for the set of all people. You can use letter F for "Bobby Fisher", and
$G(x)$: x is a chess grandmaster
$P(x)$: x lives in Philippines
$B(x, y)$: x can beat y in chess
(c) Turn the proof of part (a) into a formal proof, with references to all the rules of propositional logic you are using.

3. Prove that for all integers $n > 1$,

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}$$

4. Let $f\colon A \to B$, $g\colon B \to C$ be function such that $g \circ f\colon A \to C$ is surjective. Prove that g must be surjective. Show that f is not necessarily surjective.

5. Let $\mathbb{R}_+$ denote the set of positive real numbers. Let $f\colon \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ be given by $f(x, y) = x/y$.

(a) Is f an injective function (that is, is f one-to-one)? Prove your answer.
(b) Is f a surjection from $\mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ (that is, is it onto)? Again, prove your answer.
(c) Is f a bijection? Prove your answer.

6. Textbook p. 182, problem 1.

7. Textbook p. 155, problem 12.3.(i)

8. Textbook p. 155, problem 12.4.(i)

9. How many ways there are to select president, treasurer and secretary in a club of 16 people? (no one can occupy more than one post)

10. Prove proposition 14.2.2 in the textbook: if A, B are denumerable sets, then so is $A \cup B$.

11. (a) Let S be a denumerable set, and $s \in S$ – an element in S. Prove that the set $S - \{s\} = \{x \in S \mid x \neq s\}$ is denumerable.

(b) Sets A and B are such that $A \cup B$ is denumerable, and A is finite. Prove that then B is denumerable.

12. Textbook p. 270, problem 22.1.

13. Textbook p. 271, problem 22.3.

14. Consider the relation on $\mathbb{R}^2$ given by

$$(x, y) \Diamond (z, w) \quad \text{whenever} \quad x - y = z - w.$$

Is this an equivalence relation? Prove your answer.