

MAT 200: HOMEWORK 4
DUE TH, JUNE 30

In the problems involving real numbers, you can use all the properties of real numbers stated as axioms or proved in the textbook or in class, but nothing else.

1. Let $x > 0$. Prove that then there exists a positive integer n such that $x < n \leq x + 1$.
[Hint: apply Axiom 7.5.1 to a suitable subset of \mathbb{Z}^+ .]
2. A subset $A \subset \mathbb{R}$ is called bounded if there exists a real number M such that $|x| \leq M$ for all $x \in A$.
 - (a) Rewrite this definition using only quantifiers, logic connectives, arithmetic operations, and inequalities.
 - (b) Write a definition of unbounded set without using the word “not” (or symbol \neg).
 - (c) Show that the set of negative real numbers is unbounded.
3. P. 114, problem 9.7
4. P. 117-119, problem 13(ii)
5. P. 117-119, problem 16(ii), (iii), (vi)
6. P. 117-119, problem 18
7. P. 117-119, problem 20