MAT 200: HOMEWORK 2

DUE TU, JUNE 14

In the problems involving real numbers, you can use all the properties of real numbers stated as axioms or proved in the textbook or in class, but nothing else.

1. Let A, B, C be logical statements. Given that the following are true:

$$\begin{aligned} &A \text{ OR } B \\ &B \implies \text{NOT } C \\ &C \implies ((\text{NOT } A) \text{ OR } B) \end{aligned}$$

prove that C is false.

2. (a) Prove that $|-a| = |a|$

(b) Prove that $|a| = |b|$ if and only if $a^2 = b^2$

3. In problem 3, you need to a)write the obvious conclusion from given statements; and b)justify the conclusion, by writing a chain of arguments which leads to it. It may help to write the given statements and conclusion by logical formulas (denoting the statements which are used by letters $A, B, \dots$ connected by logical operations OR, AND, $\implies, \dots$).

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No hedgehogs can read.
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4. Use proof by contradiction to prove that there does not exist a largest positive integer.

5. Use induction to prove that every positive integer n can be written in one of the three forms: $n = 3k$ for some integer k; or $n = 3k + 1$; or $n = 3k + 2$

6. Let x be a real number, $x > -1$. Prove Bernoulli's inequality:

$$(1 + x)^n \geq 1 + nx$$

for any positive integer n.

7. Guess a formula for the product

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\dots\left(1 - \frac{1}{n^2}\right)$$

and prove it using induction