Midterm 2 Practice Problems

MAT 132

Oct 29, 2018

Name: (please print)	**ID #**:
Your recitation (e.g. R21):	(see list below)

Lecture 01	MW 5:30 PM – 6:50 PM	Benjamin Sibley
R01	TuTh 5:30pm – 6:23pm	Jean-Francois Arbour
R02	MW 12:00pm –12:53pm	John Sheridan
R03	MF 1:00pm – 1:53pm	Juan Ysimura
R04	MW 11:00am – 11:53am	John Sheridan
R05	TuTh 5:30pm – 6:23pm	Aleksandar Milivojevic
Lecture 02	MW 10:00am – 11:20am	Demetre Kazaras
R20	MW 12:00pm –12:53pm	El Mehdi Ainasse
R21	TuTh 8:30am – 9:23am	Aleksandar Milivojevic
R22	WF 4:00pm – 4:53pm	Zhaoqi Zhang
R23	MF 1:00pm – 1:53pm	Marlon de Oliveira Gomes
R24	TuTh 1:00pm – 1:53pm	Jean-Francois Arbour
R25	TuTh 5:30pm – 6:23pm	Jean-Francois Arbour
Lecture 03	TuTh 10:00am – 11:20am	Alexander Kirillov
R30	MW 9:00am – 9:53am	El Mehdi Ainasse
R31	MW 10:00am –10:53am	Timothy Alland
R32	MF 12:00pm – 12:53pm	Marlon de Oliveira Gomes
Lecture 04	TuTh 10:00am – 11:20am	Yu Li
R40	MW 9:00am – 9:53am	Timothy Alland
R41	MF 12:00pm – 12:53pm	Juan Ysimura

No notes, books or calculators.

You must show your reasoning, not just the answer. Answers without justification will get only partial credit. Your solutions should be written so that the grader is able to follow your reasoning and computations.

Please cross out anything that is not part of your solution — e.g., some preliminary computations that you didn't need. **Everything not crossed out will be considered part of your solution and graded. If the grader can't understand or follow it, points will be taken off.**

When computing numerical answers, please simplify the answers as much as possible (e.g., sin(0) should be replaced by 0). However, do not replace algebraic expressions and constants such as $\sqrt{2}$ or π by approximate values.

When computing indefinite integrals, do not forget the integration constant!

	1	2	3	4	5	6	**Total**
	15pt	20pt	15pt	15pt	20pt	15pt	100pts
Grade							

1. Compute the average value of the following functions:
(a) $f(x) = x^2 \sin(2x)$ over the interval $[-2\pi, 2\pi]$.
(b) $f(x) = e^{2x}(\cos(x) - \sin(x))$ over the interval $[-\pi, \pi]$.

2. Find a number b so that the average value of $f(x) = x^2 + x$ over the interval $[0, b]$ is equal to 4.

3. Use Euler's method to find approximate solutions of the following initial value problems:
(a) $y' = 2y - e^x$, $y(0) = 1$, step $h = 0.5$. Find y_2.
(b) $y' = \sin(x)y + 1$, $y(1) = 2$, step $h = 0.2$. Find $y(1.4)$.

4. Consider the differential equation $x^3y'' - 2x^2y' = -1$.
(a) Show that the family of functions $y(x) = c_1x^3 + c_2 - \frac{1}{4x}$ are always solutions for this differential equation.
(b) Graph the solution for which $c_1 = 1$ and $y(0.5) = 1$.
(c) Find the solution which satisfies $y(1) = 0$ and $y(2) = 1$.

5. Sketch the direction field for the differential equation

$$y' = y^2 - y - 2$$

Explain how the limiting behavior of a solution curve as $x \to \infty$ depends on the value $y(0)$.

6. Sketch the direction field for $y' = y + xy$, and a solution curve passing through $(0, 1)$.

7. Find the general solution of the following differential equation:

$$xy^2 - y'x^2 = 0$$

8. Find the solution for the equation $yy' = (y + 1)\ln(x)$ that passes through the point $(1, 2)$. [It is OK to leave teh answer in the implicit form, i.e. as an relation between x, y; you do not have to solve for y.]

9. A family of curves is obtained from a cubic parabola $y = x^3$ by translations along the y-axis. Find an equation of the orthogonal trajectories for this family. Draw both families on the same coordinate plane.

10. (a) An aquarium contains 10 gallons of polluted water. A filter drains off the polluted water at the rate of 5 gallons per hour and replaces it at the same rate by pure water. How long does it take to reduce pollution to half its initial level? (Leave your answer in terms of logarithms.)
(b) The population of California in the year 1990 was 30 million and its population in the year 2000 was 34 million. If we assume that the rate of change of the population in California is proportional to the size of its population, what will its population be in the year 2020?

11. One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction of the population who have heard the rumor and the fraction who have not heard the rumor.
(a) Write a differential equation that is satisfied by the fraction of people who have heard the rumor..

(b) Solve the differential equation.

(c) A small town has 1000 inhabitants. At 8 am, 80 people have heard a rumor. By noon half the town has heard it. At what time will 80% of the population have heard the rumor?