The midterm covers chapters 7.1-7.3 and 8.8 in the textbook. The actual exam will contain 5 problems (some multipart), so it will be shorter than this practice exam.

1. Calculate the second degree Taylor polynomial $T_2(x)$ about $a$ for the following functions.
   (a) $\sin(x^2)$ where $a = \sqrt{\pi}$.
   (b) $\arccos(x)$ where $a = 1/2$.
   (c) $x^x$ around $x = 1$.

2. Using Taylor's inequality, how well does $T_2(x)$ (calculated above) approximate $\sin(x^2)$ in the interval $[0, 2\sqrt{\pi}]$?

3. Estimate $\cos(0.1)$ to within 2 decimal places. (You may assume that the Maclaurin series for $\sin(x)$ is $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$.)

4. For which constants $b, c$ is $\sin(bx)e^{cx}$ a solution of
   (a) $y'' - 4y$.
   (b) $y'' + 2y' + 4y = 0$.

5. Draw direction fields for the following differential equations.
   (a) $y' = 1$
   (b) $y' = y$
(c) $y' = y^2 - 4$.

(d) $y' = x - y$. 
6. Use Euler's Method with step size 0.01 to estimate $y(0.02)$ where $y$ satisfies:
   (a) $y' = y, \quad y(0) = 1$. 
   (b) $y' = xy, \quad y(0) = 3$. 

7. Solve the following differential equations:
   (a) $y' = y^2, \quad y(0) = 1$. 
   (b) $y' = 1 + y^2, y(0) = 0$. 
   (c) $y' = x - y, \quad y(0) = 1$ (by substituting $u = x - y$).