

**MAT 127, MIDTERM 2
PRACTICE PROBLEMS**

The midterm covers chapters 7.1-7.3 and 8.8 in the textbook. The actual exam will contain 5 problems (some multipart), so it will be shorter than this practice exam.

1. Calculate the second degree Taylor polynomial $T_2(x)$ about a for the following functions.

- (a) $\sin(x^2)$ where $a = \sqrt{\pi}$.
- (b) $\arccos(x)$ where $a = 1/2$.
- (c) x^x around $x = 1$.

2. Using Taylor's inequality, how well does $T_2(x)$ (calculated above) approximate $\sin(x^2)$ in the interval $[0, 2\sqrt{\pi}]$?

3. Estimate $\cos(0.1)$ to within 2 decimal places. (You may assume that the Maclaurin series for $\sin(x)$ is $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$.)

4. For which constants b, c is $\sin(bx)e^{cx}$ a solution of
- (a)

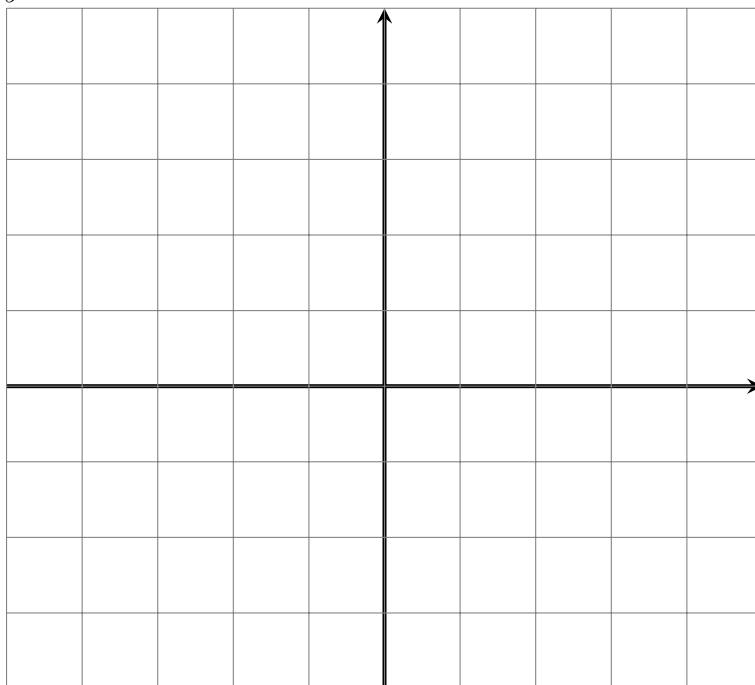
$$y'' - 4y.$$

- (b)

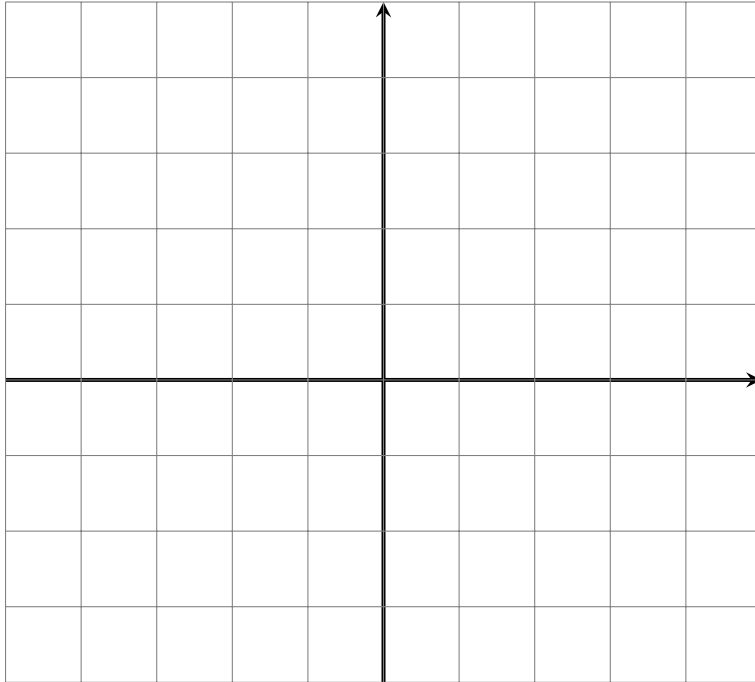
$$y'' + 2y' + 4y = 0.$$

5. Draw direction fields for the following differential equations.

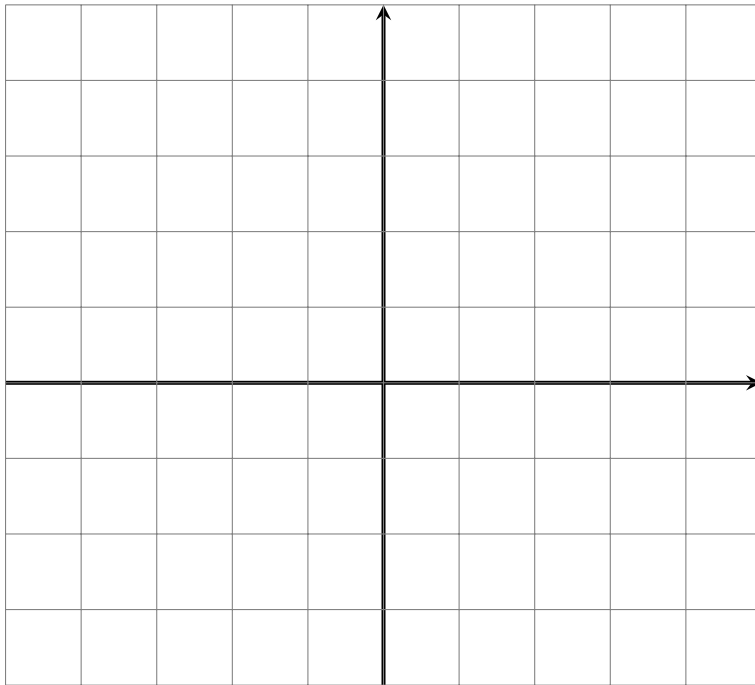
- (a) $y' = 1$



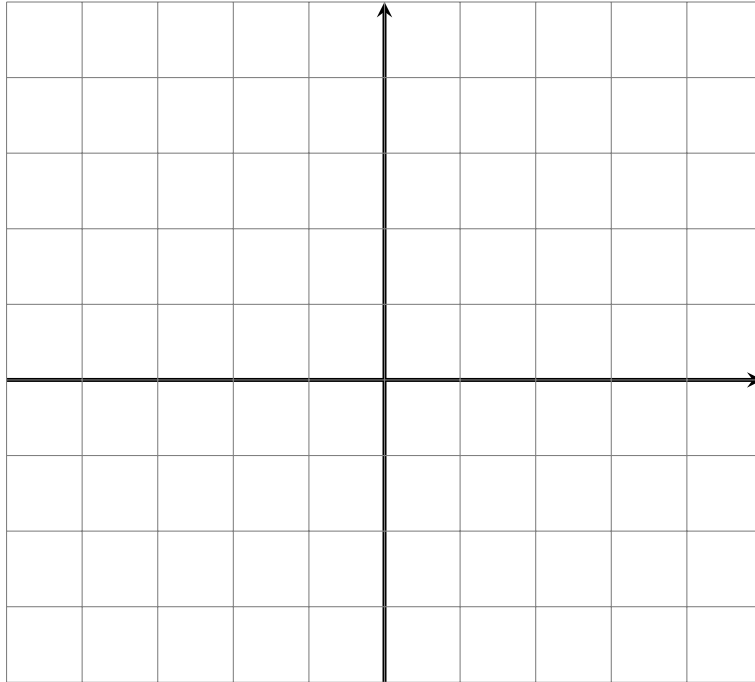
- (b) $y' = y$



(c) $y' = y^2 - 4$.



(d) $y' = x - y$.



6. Use Euler's Method with step size 0.01 to estimate $y(0.02)$ where y satisfies:
- (a) $y' = y, \quad y(0) = 1.$
 - (b) $y' = xy, \quad y(0) = 3.$
7. Solve the following differential equations:
- (a) $y' = y^2, \quad y(0) = 1.$
 - (b) $y' = 1 + y^2, \quad y(0) = 0.$
 - (c) $y' = x - y, \quad y(0) = 1$ (by substituting $u = x - y$).