MAT126 Fall 2009 Practice Midterm I

The exam will be a closed notes and closed book. The use of electronic devices will be not permitted. Once the exam has begun, having notes or books on the desk or in view will be considered cheating and will be referred to the Academic Judiciary.

The actual midterm will contain six problems.

Problem 1 Evaluate the integral by interpreting it as an area:

$$\int_0^3 f(x)dx,$$

where

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 1, \\ 1 & \text{if } 1 \le x \le 3. \end{cases}$$

Problem 2 Evaluate

$$\lim_{n \to \infty} \sum_{i=1}^n \left(4 + \frac{5i}{n} \right)^{1/2} \frac{5}{n}$$

by interpreting the limit as an integral and using the Evaluation Theorem to compute this integral.

Problem 3 Evaluate the following definite integral:

$$\int_{2}^{4} \frac{x^3 \sqrt{x^5} - x^2 \sqrt[3]{x^2}}{x^4} dx$$

Problem 4 Let f(x) and g(x) be two functions such that

$$\int_{-1}^{2} [f(x) + g(x)]dx = 3, \\ \int_{-1}^{2} [f(x) - 2g(x)]dx = 1, \\ \int_{-1}^{0} f(x)dx = -1$$

Find $\int_0^2 f(x) dx$

Problem 5 1. Find $\frac{d}{dx}(e^{-x^2})$.

2. Evaluate

$$\int_0^2 x e^{-x^2} dx.$$

Problem 6 Find antiderivative of

1.
$$f(x) = \frac{\sin(2x)}{\cos(x)}$$
 (Hint:use trigonometric identity)
2. $f(x) = e^{x+7}2^{-2x}$
3. $f(x) = \frac{x^2}{x^3}$

Problem 7 Compute the derivative of the function

 $\ln(\tan^2(x))$

and simplify your answer.

Problem 8 Estimate the integral

$$\int_{1}^{4} x \ln x dx$$

using three rectangles and

a) right endpoints b) left endpoints.

c) Are your answers in a) and b) over- or under-estimates of the actual integral?

(Hint: you may want to determine whether the function $f(x) = x \ln x$ is increasing or decreasing)