Please answer each question in the space provided. Please write full solutions, not just answers. Unless otherwise marked, answers without justification will get little or no partial credit. Cross out anything the grader should ignore and circle or box the final answer. Do NOT round answers.

No books, notes, or calculators!

Do not open the exam until instructed by proctor!
1. Compute the following limits. Please distinguish between “limit is equal to $\infty$”, “limit is equal to $-\infty$” and “the limit doesn’t exist even allowing for infinite values”:

(a) \( \lim_{x \to \infty} \frac{x^3 + 2x + 1}{x^3 - 15x} \)

(b) \( \lim_{x \to -2} \frac{x^2 - 2x - 3}{x^2 - 5x + 6} \)

(c) \( \lim_{x \to 3^+} \frac{x^2 - 2x - 3}{x^2 - 5x + 6} \)

(d) \( \lim_{x \to \infty} \frac{1}{e^{(x^2)} + 1} \)

2. Calculate derivatives of the following functions:

(a) \( 3(x + \sqrt{x}) \)

(b) \( x e^x - 17x \)

(c) \( \frac{2x}{x+1} \)

(d) \( \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \)

3. (15 points) Let \( f(x) = |1 + \frac{1}{x}| \).

(a) Sketch the graph of \( f \) and identify the asymptotes.

(b) Find all values of \( x \) for which \( f \) is not continuous.

(c) Find all values of \( x \) for which \( f \) is not differentiable (you do not have to calculate the derivative).
4. Match the graphs of functions I–IV below with the graphs of their derivatives A–D. (Justification is not required.)
5. Let \( f(x) = x^3 - 3x^2 - 9x + 7 \).

(a) Calculate \( f' \)

(b) Calculate \( f'' \)

(c) On which intervals does \( f \) increase? decrease?

(d) On which intervals is \( f \) concave up?

6. Find all tangent lines to the graph of \( f(x) = 1/x \) which have slope \( m = -1/4 \); write equations of each of these tangent lines.

7. (a) Write the linear approximation for the function \( g(x) = \frac{1}{e^x + 1} \) near \( x = 0 \).

(b) Use the linear approximation you found in the previous part to estimate \( \frac{1}{e^m + 1} \).