Review Sheet: Test 2 (Solutions)

1.

$$a)\ y_1 = e^t, \quad y_2 = e^{-2t}, \quad y_3 = e^{-t}, \quad y = c_1y_1 + c_2y_2 + c_3y_3,$$
$$b)\ y_1 = e^{3t}, \quad y_2 = te^{3t}, \quad y = c_1y_1 + c_2y_2.$$

2.

$$a)\ y_p = -\frac{1}{5}xe^{-x} + \frac{1}{13}e^x[10\cos 2x + 2\sin 2x],$$
$$b)\ y_p = -\frac{3}{2}x\cos 2x + \frac{3}{4}(\sin 2x)\log(\sin 2x).$$

3. Frequency= $\sqrt{128}$ oscillations per second, Period= $2\pi/\sqrt{128}$ seconds per oscillation, Amplitude= $\sqrt{\frac{1}{9} + \frac{4}{128}}$ feet, Phase= $\tan^{-1}(-6/\sqrt{128})$.

4. b) 7.33 miles.

5.

$$a)\ x_p = -\frac{1}{2} + \frac{16}{\sqrt{(64-4)^2+4}}\cos(2t-\delta) \quad \text{where} \quad \delta = \tan^{-1}\frac{1}{30},$$
$$b)\ m = 4.$$

6. Use direct substitution.

7.

$$\vec{x}' = \begin{pmatrix} 0 & 1 \\ t^{-2}\sin t & -t^{-1} \end{pmatrix}\vec{x} + \begin{pmatrix} 0 \\ t^{-2}e^t \end{pmatrix}, \quad \vec{x}(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

8. See back of book.

9. See back of book.

10.

$$\begin{aligned} m_1x_1'' &= -m_1g - k_1x_1 + k_2(x_2 - x_1) \\ m_2x_2'' &= -m_2g - k_2(x_2 - x_1). \end{aligned}$$

11. a) $W = t^2$, b) all intervals, c) the coefficients are not continuous across zero,

$$d)\ \vec{x}' = P(t)\vec{x}, \quad P(t) = \begin{pmatrix} 0 & 1 \\ -2t^{-2} & 2t^{-1} \end{pmatrix}.$$

12. If x_p is a particular solution and x is any solution of the nonhomogeneous equation, then $x - x_p$ solves the homogeneous equation. By uniqueness $x - x_p = x_h$.

13.

$$\vec{x}(t) = c_1e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2e^{-2t}\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \lim_{t\to\infty}\vec{x}(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

14.

$$\overrightarrow{x}(t) = 2e^{t}\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} - 8e^{2t}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 6e^{3t}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}.$$