Review Sheet: Test 2

The test will cover sections 3.1-3.6 as well as 5.1 and 5.2 from the book, in addition to all material covered in the lectures between the last test and the current date (excluding sections 5.3-5.6 and the material on complex and repeated eigenvalues). You may bring one sheet of paper (8.5in × 11in) to the test, on which you may record any helpful formulas, problems, etc... However, calculators and other computing devices will not be permitted. The following list of problems will help in preparing for the exam.

1. Find a fundamental set of solutions, and write down the general solution of:

$$a)\ y^{(3)} + 2y'' - y' - 2y = 0, \qquad b)\ y'' - 6y' + 9y = 0.$$

2. Find a particular solution of:

$$a)\ y'' - 3y' - 4y = e^{-x} - 8e^{x}\cos 2x, \qquad b)\ y'' + 4y = 3\csc 2x, \quad 0 < x < \frac{\pi}{2}.$$

3. A mass weighing 3 lb. stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in., and then set in motion with a downward velocity of 2 ft./sec., and if there is no air resistance, find the position of the mass at any time t. Determine the frequency, period, amplitude, and phase of the motion.

4. Problems 5 and 6 in section 3.4.

5. A spring is stretched 6 in. by a mass that weighs 8 lb. The mass is attached to a dashpot mechanism that has a damping constant of 0.25 lb.-sec./ft. and is acted upon by an external force of $4\cos 2t$ lb. a) Determine the steady state response of this system. b) If the given mass is replaced by a mass m, determine the value of m for which the practical resonance is greatest.

6. Problem 29 in section 3.6.

7. Write the given initial value problem as a first order system:

$$t^2 y'' + ty' - (\sin t)y = e^{t}, \qquad y(0) = 2, \qquad y'(0) = 0.$$

8. Problem 31 in section 5.2.

9. Problem 35 in section 5.2.

10. Suppose that a sequence of two masses m_1 and m_2 are attached to a ceiling with a sequence of two springs having spring constants k_1 and k_2. Write down the equations of motion for the three masses.

11. Consider the vectors

$$\vec{x}_1(t) = \begin{pmatrix} t \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{x}_2(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}.$$

a) Compute the Wronskian. b) In what intervals are the two vectors linearly independent? c) What conclusion can be drawn about the coefficients in the system of homogeneous differential equations satisfied by $\overrightarrow{x}_1$ and $\overrightarrow{x}_2$? d) Find this system of equations and verify the conclusions of part (c).

12. Show that the general solution of the system

$$\overrightarrow{x}' = P(t)\overrightarrow{x} + \overrightarrow{g}(t)$$

is the sum of any particular solution and the general solution of the corresponding homogeneous equation.

13. Find the general solution of the given system of equations. Sketch a few of the trajectories and describe the behavior of the solutions as $t \to \infty$:

$$\overrightarrow{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \overrightarrow{x}.$$

14. Solve the given initial value problem and describe the behavior of the solution as $t \to \infty$:

$$\overrightarrow{x}' = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix} \overrightarrow{x}, \quad \overrightarrow{x}(0) = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}.$$