MAT 127 Midterm II

April 1, 2015 8:45-10:15pm

Name:____________________________ **ID:**__________________

first name *first*

Section:	L1	L2	L3	L4	L5	(circle yours)
	MWF 10-10:52am	MW 4-5:20pm	MWF 11-11:53am	TuTh 8-9:20am	TuTh 4-5:20pm	

DO NOT OPEN THIS EXAM YET

Instructions

(1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.

(4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.

(5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.

(6) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. 1/2, not 2/4) to receive full credit.

(7) Show your work; correct answers alone will receive only partial credit (unless noted otherwise).

Out of fairness to others, please **stop working** and close the exam as soon as the time is called.

Do not write in the boxes at the bottom of this page.

1 (15pts)	2 (20pts)	3 (20pts)	4 (10pts)	5 (15pts)	6 (20pts)	Tot (100pts)

Problem 1 (15pts)

(a; 7pts) Show that the function $y(x) = -e^{-x}\sin x$ is a solution to the initial-value problem

$$y'' + 2y' + 2y = 0, \quad y = y(x), \qquad y(0) = 0, \qquad y'(0) = -1.$$

Show your work and/or explain your reasoning.

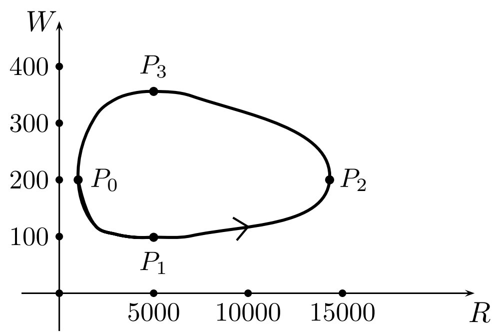
(b; 8pts) Find the general solution of the differential equation

$$y'' + 2y' + 2y = 0, \quad y = y(x).$$

Show your work and/or explain your reasoning.

Problem 2 (20pts)

The graph on the left shows a trajectory in the phase plane for the predator-prey model described by the system of ODEs on the right. R denotes the number of rabbits and W denotes the number of wolves. Initially (at time $t=0$), $R=1000$ and $W=200$.



$$\begin{cases} \frac{\mathrm{d}R}{\mathrm{d}t} = R - \frac{1}{200}RW \\ \frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{10}W + \frac{1}{50{,}000}RW \end{cases}$$

(a; 10pts) Sketch a rough graph of R as a function of $t = \mathit{time}$.

(b; 2pts) When the number of rabbits reaches its global maximum, about how many wolves are there? *Answer only.*

(c; 2pts) When the number of rabbits reaches its global maximum, is the wolf population increasing or decreasing? *Answer only.*

(d; 6pts) Find the equilibrium solutions of the system of ODEs.

Problem 3 (20pts)

Write your answer to each question in the corresponding box *in the simplest possible form.* Justify your answers in the spaces below.

(a; 7pts) Find the limit of the sequence $a_n = n^2\big(1-\cos(1/n)\big)$.

(b; 7pts) Find the limit of the sequence

$$\sqrt{2},\ \sqrt{2\sqrt{2}},\ \sqrt{2\sqrt{2\sqrt{2}}},\ \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}},\ \ldots$$

(c; 6pts) Write the number $1.0\overline{50} = 1.0505050\ldots$ as a simple fraction.

Problem 4 (10pts)

Determine whether each of the following sequences converges or diverges. In each case, *circle* your answer to the right of the question and *justify* it in the space provided below the question. If the sequence converges, find its limit.

(a; 5pts) $a_n = \dfrac{(-1)^n n^3}{n^3 + 2n^2 + 1}$ **converge** **diverge**

(b; 5pts) $\left\{1, 1 + \dfrac{1}{2}, 1 + \dfrac{1}{2} + \dfrac{1}{3}, 1 + \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4}, \dots\right\}$ **converge** **diverge**

Problem 5 (15pts)

(a; 7pts) Determine whether the following series **converges** or **diverges**

$$\sum_{n=1}^{\infty} ne^{-n}$$

Circle your answer above and justify it below.

(b; 8pts) Find all values of p for which the following series converges

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}.$$

Write your answer in the box to the right and justify it below.

$\boxed{}$

Problem 6 (20pts)

For each of the following series, determine whether it converges and if so, find its sum. Simplify your answers as much as possible and justify them.

(a; 8pts) $\displaystyle\sum_{n=1}^{\infty} \frac{1+4^n}{1+3^n}$

(b; 12pts) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

Hint: partial fractions.