GEOMETRY AND MATHEMATICAL PHYSICS WORKSHOP DEDICATED TO THE MEMORY OF VASIL TSANOV

1. Titles and abstracts

1.1. Bogdan Alexandrov.

Title: *Remarks on homogeneous 3-Sasakian manifolds*

Abstract: A Riemannian manifold is called 3-Sasakian if the cone over it is hyper-Kähler. Each 3-Sasakian metric determines a natural 1-parameter family of Riemannian metrics of which it is a member. The homogeneous 3-Sasakian manifolds are known. Examples are the sphere and the real projective space of dimension 4n + 3. It is mentioned in passing in some works of Boyer and Galicki that these examples are the only naturally reductive homogeneous 3-Sasakian manifolds. In my talk I will show that for each homogeneous 3-Sasakian manifold the whole 1-parameter family consists of naturally reductive metrics (which, in particular, corrects the above statement). I will also determine which metrics of the 1-parameter family satisfy the more restrictive condition of being normal homogeneous. Similar results hold true for 3-Sasakian manifolds of signature (3, 4n). I will add some further remarks if time allows.

1.2. Lyubomir Andreev.

Title: *Gromov hyperbolicity for the Kobayashi distance*

Abstract: The notion of hyperbolicity for a metric space introduced by M. Gromov generalizes many of the global features of the geometry of complete, simply connected manifolds of negative curvature. There are extensive studies of interesting classes of Gromov hyperbolic spaces in the literature, the class of word-hyperbolic discrete groups perhaps being the most studied. In a different direction, domains in Euclidean spaces endowed with distances arising from certain natural Finsler metrics have also been analyzed from this point of view. In several complex variables there are studies on Gromov hyperbolicity for smooth bounded pseudoconvex domains with respect to biholomorphically invariant distances. In the present talk we will focus on Gromov hyperbolicity of bounded strictly pseudoconvex domains endowed with the Kobayashi distance.

1.3. Khristo Boyadzhiev.

Title: *Recollections about Vasil and some notes on classical analysis*

Abstract: The talk starts with recollections about younger years of Vasil Tsanov (student years, Ph. D student, and assistant professor). This will be followed by some historical notes and a discussion about trends and fashions in mathematics.

Next we review series of the form $\sum_{n=0}^{\infty} a_n b_n x^n$ where a_n and b_n are various special numbers (as harmonic numbers, binomial coefficients, Catalan numbers, Stirling numbers, Bernoulli numbers, zeta values, etc). The above series is the Hadamard product of the two power series $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$. We also discuss Euler's series transformation and the related binomial transform of sequences.

1.4. **Danail Brezov.**

Title: *Projective View on Motion Groups*

Abstract: This is an attempt for a consistent pedagogical study on the generalized Rodrigues' projective construction of low-dimensional motion groups starting with SO(3) and then gradually extending to the Galilean and Lorentzian settings. In the case of spatial rotations one encounters $\mathbb{RP}^3$ with an additional group structure inherited from quaternion multiplication, which allows for associating particular types of curves in $\mathbb{E}^3$ with rigid body kinematics, based on the corresponding Maurer–Cartan form. A similar construction in complex projective space yields the relativistic version of the above approach, while a dual extension leads respectively to the study of nonhomogeneous isometries. There are also further generalizations related to conformal models, discussed in this context.

1.5. **George Dimitrov.**

Title: *Non-commutative counting invariants*

Abstract: The concept of *counting* is among the oldest in Mathematics. The notion of category was introduced in 20 century and it is no less important (for mathematical Physics as well). Grothendieck, Bondal, Kontsevich et al. used the latter to invent *non-commutative* algebraic geometry.

Guideline in my recent work is what I call non-commutative counting. Viewing certain triangulated categories as non-commutative curves, I observed that it is reasonable to count non-commutative curves in triangulated categories. From this idea arise new categorical invariants.

The talk is a sketch of the definition of these invariants, structures on them, and functoriality. Non-commutative counting will be exemplified by producing many finite numbers, and by relating it to combinatorics, classical geometry and number theory. Intersection of the entities I count will be illustrated, time permitting. The talk is based on a joint work with L. Katzarkov.

1.6. **Ivan Dimitrov.**

Title: *Left symmetric superalgebras*

Abstract: Left symmetric algebras (LSA's) are the algebraic counterpart of flat left-invariant connections on differentiable manifolds. Similar to associative algebras, LSA's give rise to Lie algebras and it is a natural question to ask whether a given Lie algebra arises from a LSA. It turns out that the answer is rarely positive - it is negative for semisimple Lie algebras; a full classification of the LSA-structures on gl_n was provided by O. Baues.

In my talk I will explain the necessary background and Baues' result. Then I will discuss the classification result about basic classical Lie superalgebras. As a teaser, the superalgebras $sl(m+1|m)$ admit an interesting LSSA-structure.

This is a joint work with Runxuan Zhang.

1.7. **Penka Georgieva.** Title: *A Klein TQFT : the local real Gromov-Witten theory of curves*

Abstract: The local Gromov-Witten theory of curves studied by Bryan and Pandharipande revealed strong structural results for the local GW invariants, which were later used by Ionel and Parker in the proof of the Gopakumar-Vafa conjecture. In this talk I will report on a joint work with Eleny Ionel on the extension of these results to the real setting. We show that the local real GW theory gives rise to a 2-dimensional Klein TQFT defined on an extension of the category of unorientable surfaces. We use this structure to completely solve the theory by providing a closed formula for the local real GW invariants in terms of representation theoretic data. As a corollary we obtain the local version of the real Gopakumar-Vafa formula. In the case of the resolved conifold the partition function of the real GW invariants agrees with that of the SO/Sp Chern-Simons theory on S^3.

1.8. **Dimitar Grantcharov.**

Title: *Relations between Laplace spectra and geometric quantizaion of Reimannian symmetric spaces*

Abstract: In this talk we will discuss a geometric quantization scheme due to Kostant-Souriau and Czyz-Hess. This scheme is applied to the cotangent bundles of compact rank-one Riemannian symmetric spaces (CROSS). It turns out that the energy spectrum obtained from the quantization is related to the spectrum of the Laplace-Beltrami operator of the CROSS. Moreover, the real dimensions of the corresponding eigenspaces coincide with the complex dimensions of the spaces of holomorphic sections of the quantum bundles. The relation between the two constructions was first noticed by Mladenov and Tsanov for the case of the spheres. This is a joint work with Gueo Grantcharov.

1.9. **Mihail Hamamdjiev.**

Title: *Multidirectional mean value inequalities without compact requirements*

Abstract: In this talk we are inspired by the (now) classical work (in the field) of Clarke and Ledyaev on multidirectional mean value inequalities of type set-set to apply a recent result of Hamamdjiev and Ivanov, which uses a general type of subdifferential and to consider multidirectional inequalities without compactness restrictions we obtain in this way.

1.10. **Elitza Hristova.**

Title: *Hilbert series and invariants of symplectic and orthogonal groups*

Abstract: Let $A = \bigoplus_{i\geq 0} A^i$ be a finitely generated graded algebra over $\mathbb{C}$ such that each homogeneous component A^i is a polynomial $\mathrm{GL}(n,\mathbb{C})$-module. Examples of such algebras are the symmetric algebra $S(W)$ and the exterior algebra $\Lambda(W)$ of a polynomial $\mathrm{GL}(n,\mathbb{C})$-module W and more generally, any algebra $T(W)/I$, where $T(W)$ denotes the tensor algebra of W and I is a $\mathrm{GL}(n,\mathbb{C})$-invariant ideal in $T(W)$. In this talk, we consider such an algebra A and describe a method for determining the Hilbert series of the algebra of invariants A^G, where G is one of the classical groups $\mathrm{O}(n,\mathbb{C})$, $\mathrm{SO}(n,\mathbb{C})$ and $\mathrm{Sp}(2k,\mathbb{C})$ (in the case when $n = 2k$). This method extends previous results for the case $G = \mathrm{SL}(n,\mathbb{C})$. Then we give concrete examples in which the Hilbert series can be computed explicitly. In some of the examples, using the computed Hilbert series we give sets of generators for the respective algebras of invariants. The talk is based on a joint work with Vesselin Drensky.

1.11. **Stefan Ivanov.** Title: *Quaternionic Heisenberg group and Solutions to Strominger system with non-constant dilaton in small dimensions*

Abstract: Solutions to the Strominger system with non trivial 3-form fluxes, non-trivial instanton and non constant dilaton is presented. The dilaton depends on one variable and is determined as a real slice of the Weierstrass' elliptic function. These supersymmetric solutions solve also the heterotic equations of motion (generalized heterotic string version of the Einstein vacuum equations of general relativity) up to the first order of the string tension α'.

1.12. **Ludmil Katzarkov.**

Title: *New birational invariants*

Abstract: We will define new birational invariants coming from PDE and simplectic geometry. This is a joint work with M. Kontsevich and T. Pantev.

1.13. **Tony Pantev.**

Title: *Universal constructions of symplectic structures*

Abstract: I will discuss the notion of relative shifted symplectic structures on sheaves of derived stacks over locally compact Hausdorff topological spaces. I will describe a general pushforward construction of relative symplectic forms and in the constructible case will explain explicit techniques for computing such forms. As applications I will discuss extensions of Hamiltonian and quasi-Hamiltonian reduction, symplectic leaves in the moduli of local systems on higher dimensional varieties, and a universal construction of symplectic structures on derived irregular character varieties. This is a joint work with Dima Arinkin and Bertrand Toen.

1.14. **Ulf Persson.**

Title: *Mathematical Platonism*

Abstract: This is a philosophical talk arguing for the reality of mathematics. It will start out in general philosophical terms trying to clarify what we mean by reality. The classical axiomatic approach is discussed in particular compared with the natural sciences. Finally it is shown why it is unsatisfactory and does not cover the nature of mathematics which cannot be reduced to logic.

1.15. **Ivan Todorov.**

Title: *Exceptional quantum algebra for the standard model of particle physics*

Abstract: The exceptional euclidean Jordan algebra J_3^8 , consisting of 3×3 hermitian octonion matrices, appears to be tailor made for the internal space of the three generations of quarks and leptons. The maximal rank subgroup of the automorphism group F_4 of J_3^8 that respects the lepton-quark splitting is $SU(3)_c \times SU(3)_{ew}/\mathbb{Z}_3$. Its restriction to the special Jordan subalgebra $J_2^8 \subset J_3^8$, associated with a single generation of fundamental fermions, is precisely the symmetry group $S(U(3) \times U(2))$ of the Standard Model. The Euclidean extension $\mathcal{H}_{16}(\mathbb{C}) \otimes \mathcal{H}_{16}(\mathbb{C})$ of J_2^8 , the subalgebra of hermitian matrices of the associative envelope of its complexification, involves 32 primitive idempotents giving the states of the first generation fermions. The triality relating left and right $Spin(8)$ spinors to 8-vectors corresponds to the Yukawa coupling of the Higgs boson to quarks and leptons.

The talk is based on the paper https://arxiv.org/abs/1604.01247v2 of Michel Dubois-Violette and on ongoing work with him and with Svetla Drenska: https://arxiv.org/abs/1806.09450,

https://arxiv.org/abs/1805.06739v2, https://arxiv.org/abs/1808.08110.

1.16. **Valdemar Tsanov.**

Title: *Homogeneous projective varieties and invariant rings*

Abstract: A classical theorem of Hilbert asserts that the ring of invariant polynomials on a linear representation V of a reductive complex Lie group G is finitely generated. However, the proof is nonconstructive, and this nonconstructiveness remains a source of unsovled problems in representation theory. I will discuss some recent results on the properties of generating invariants, in particular their degrees, related to the geometry of closed G-orbits in the projective space of the representation. The key roles are played by the secant varieties of the closed orbits and the unstable locus - the common vanishing locus of all nonconstant homogeneous invariants.

1.17. **Stephan Zhechev.**

Title: *Embeddability is undecidable outside the meta-stable range*

Abstract: We will prove that the following question is algorithmically undecidable for $k+2 < d < 3(k+1)/2$, and $k > 5$, which covers essentially everything outside the meta-stable range: Given a finite simplicial complex K of dimension k and an embedding $f : L \to \mathbb{R}^d$ of a subcomplex L of K, can f be extended to an embedding $F : K \to \mathbb{R}^d$ of the whole complex? Here, we assume that the given embedding f of the subcomplex L is linear (on each simplex of L) whereas the desired embedding F of the whole complex is allowed to be piecewise-linear (i.e., linear on an arbitrarily fine subdivision of K); moreover F is not required to be part of the output.

More generally, we prove that the question of deciding embeddability, which is the special case of the question above when we set L to be empty, is also undecidable for $k + 2 < d < 3(k + 1)/2$, and $k > 5$.

The strategy of our proof is to construct a reduction from Hilbert's tenth problem to both the embeddability and extension of embeddings problems. More specifically, for particular types of systems of quadratic Diophantine equations, we will show how to construct corresponding instances of the two problems we consider, so that an embedding or extension exists if and only if the original system of equations has an integer solution. This is a joint work with Uli Wagner and Marek Filakovsky.