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MAT319/320 Zhao/Schul Midterm 1 September 21, 2020

First Name: ____________________

Last Name: ____________________

Stony Brook ID: ____________________

Signature: ____________________

Write coherent mathematical statements and show your work on all problems. If you use a theorem from the book, you must fully state it. If you give an example then you must prove it is such. Please write clearly.

Rules.

(1) Start when told to; stop when told to.
(2) No notes, books, etc,...
(3) Turn OFF all unauthorized electronic devices (for example, your cell phone).
(4) You may NOT collaborate with any person. Further, you may NOT communicate with anyone about the contents of this exam for **2 days (48 hours)**. Any work that you submit must be your OWN.

1 (10pts)	2 (10pts)	3 (10pts)	4 (10pts)		TOTAL

QUESTIONS

(1) Let $(u_n)_{n\in\mathbb{N}}$ be a sequence of real numbers defined by $u_1 = 1$ and
$$u_{n+1} = \sqrt{2+u_n} \text{ for any } n \in \mathbb{N}.$$
Prove by induction that $0 < u_n < 2$ for any $n \in \mathbb{N}$. Note: for this question you may assume that for any $a \in \mathbb{R}$ with $0 \leq a$ there is a number $\sqrt{a} \in \mathbb{R}$ such that $0 \leq a$ and $\sqrt{a} \cdot \sqrt{a} = a$.

(2) Recall one definition of supremum. Prove that if a subset of $\mathbb{R}$ has a supremum then the supremum is unique, i.e. for $A \subset \mathbb{R}$ there can be at most one $a \in \mathbb{R}$ such that $a = \sup(A)$.

(3) Prove the following theorem:

Let $[a_n, b_n], n \in \mathbb{N}$ be a nested sequence of bounded closed intervals. Then

$$\cap_{n\in\mathbb{N}}[a_n, b_n] \neq \emptyset.$$

(4) Let $(u_n)_{n\in\mathbb{N}}$ be a sequence of positive real numbers such that $u_n < \frac{1}{k} + \frac{k}{n}$ for any $k, n \in \mathbb{N}$. Prove that $\lim_{n\to+\infty} u_n = 0$. Hint: use the definition of a limit.

Note: Partial credit will be given for assuming that $u = \lim_{n\to+\infty} u_n$ exists, and then showing that in that case $u = 0$.

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