MAT535 HOMEWORK 9

Due on May 3rd in class. Each problem is worth 10 points. You are required to do five of the problems of your choice. You can do the rest for extra credit. Note that we denote algebraic sets by $V(S)$ instead of $Z(S)$ from the book [DF].

Problem 1. Show that $I(\mathbb{A}^n) = (0)$.

Problem 2. If $I \subset R$ is any ideal, show that $\sqrt{I}$ is a radical ideal.

Problem 3.

(a) $S \subset I(V(S))$.

(b) $W \subset V(I(W))$.

(c) If W is an algebraic set then $W = V(I(W))$.

(d) If $I \subset k[x_1, \dots, x_n]$ is any ideal then $V(I) = V(\sqrt{I})$ and $\sqrt{I} \subset I(V(I))$.

Problem 4.

(a) Show that the set $X = \{(t, t^2, t^3) \in \mathbb{A}^3 | t \in k\}$ is closed in $\mathbb{A}^3$ and find $I(X)$.

(b) Same for the subset $Y = \{(t^3, t^4, t^5) \in \mathbb{A}^3 | t \in k\}$ of $\mathbb{A}^3$.

(c) Show that $I(Y)$ can't be generated by less than three polynomials.
Hint: Is $I(Y)$ a graded ideal ?

Problem 5. Show that $W = \{(x, y, z) \in \mathbb{A}^3 | x^2 = y^3, y^2 = z^3\}$ is an irreducible closed subset of $\mathbb{A}^3$ and find $I(W)$.

Problem 6. Find $\sqrt{(y^2 + 2xy^2 + x^2 - x^4, x^2 - x^3)}$.

Problem 7. Let X be a Noetherian topological space.

(a) If an irreducible closed set Y is contained in a union $\cup X_i$ of finitely many closed sets X_i, then $Y \subset X_i$ for some i.

(b) X has finitely many components.

(c) X is the union of its components.

(d) X is not the union of any proper subset of its components.