Problem 1. (a) Let $X$ be an affine variety, $M$ a $k[X]$-module, and $\mathcal{F}$ and $\mathcal{O}_X$-module. Show that $\text{Hom}_{k[X]}(M, \Gamma(X, \mathcal{F})) \cong \text{Hom}_{\mathcal{O}_X}(\tilde{M}, \mathcal{F})$;

(b) If $X$ is an affine variety and $M$ and $N$ are $k[X]$-modules, then $\tilde{M} \otimes_{k[X]} \tilde{N}$;

(c) If $f \in k[X]$, then $\tilde{M}|_{D(f)} = \tilde{M}_f$;

(d) If $f : X \to Y$ is a morphism of varieties and $\mathcal{G}$ is a (quasi-)coherent $\mathcal{O}_Y$-module, then $f^*\mathcal{G}$ is a (quasi-)coherent $\mathcal{O}_X$-module.

Problem 2. (a) Let $X$ be a ringed space, $\mathcal{F}$ and $\mathcal{G}$ are $\mathcal{O}_X$-modules. Prove that the assignment: $U \to \text{Hom}_{\mathcal{O}_U}(\mathcal{F}|_U, \mathcal{G}|_U)$ defines an $\mathcal{O}_X$-module. It is denoted by $\text{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{G})$.

(b) Let $\mathcal{L}$ be an invertible $\mathcal{O}_X$-module. Show that $\mathcal{L}^{-1} = \mathcal{H}_{\text{Hom}_{\mathcal{O}_X}(\mathcal{L}, \mathcal{O}_X)}$ is also invertible and that $\mathcal{L}^{-1} \otimes_{\mathcal{O}_X} \mathcal{L} \cong \mathcal{O}_X$.

Problem 3. Let $X$ be a scheme of characteristic $p > 0$, $F : X \to X$ the Frobenius morphism, and $\mathcal{L}$ an invertible $\mathcal{O}_X$-module. Show that $F^*\mathcal{L} \cong \mathcal{L}^{\otimes p}$.

Problem 4. A morphism $f : X \to Y$ of varieties is called affine if for every open affine set $V \subset Y$, the inverse image $f^{-1}(V)$ is also affine. $f$ is called finite if it is affine and $k[f^{-1}(V)]$ is a finitely generated $k[V]$-module for all open affine $V \subset Y$. Let $Y = \bigcup V_i$ be an open affine covering of $Y$ such that $f^{-1}(V_i)$ is affine for every $i$. Show that $f$ is affine. If, moreover, $k[f^{-1}(V_i)]$ is a finitely generated $k[V_i]$-module for all $i$ then $f$ is finite.

Problem 5. (a) Let $X$ be a projective variety and $f : X \to Y = \text{Spec} - m(k)$ the unique morphism to a point. Show that $f^* : \mathcal{O}_Y \to f_*\mathcal{O}_X$ is an isomorphism.

(b) Find a projective variety $X$ and a birational morphism $f : X \to Y$ such that $f_*\mathcal{O}_X$ is not locally free on $Y$. 

Due 12/8 in class. Each problem is worth 10 points