MAT536 HW8

Due 11/19 in class. Each problem is worth 10 points

Problem 1. Let $\mathcal{F}$ be a sheaf on X and $p \in X$ a point. Prove the following from the definition of the stalk $\mathcal{F}_p$:
(a) Each element of $\mathcal{F}_p$ has the form s_p for some section $s \in \mathcal{F}(U), p \in U$.
(b) Let $s \in \mathcal{F}(U), p \in U$. Then $s_p = 0$ iff $s|_V = 0$ for some $p \in V \subset U$.
(c) Let $s \in \mathcal{F}(U)$. Prove that $s = 0$ iff $s_p = 0$ for every $p \in U$.

Problem 2. Let $\phi : \mathcal{F} \to \mathcal{G}$ be a morphism of sheaves on X. Show that ϕ is surjective if and only if the following holds: for every open set $U \subset X$, and for every $s \in \mathcal{G}(U)$, there is a covering $U = \cup V_i$ of U and sections $t_i \in \mathcal{F}(V_i)$ such that $\phi_{V_i}(t_i) = s|_{V_i}$ for all i.

Problem 3. Let $\mathcal{F}$ be a sheaf on X and $s \in \mathcal{F}(X)$ a global section. Show that the set $\{p \in X | s_p \neq 0\}$ is a closed subset of X.

Problem 4. Let $\phi : \mathcal{F} \to \mathcal{G}$ be a morphism of sheaves on X. Show that $Ker(\phi)_p = Ker(\phi_p)$ and $Im(\phi)_p = Im(\phi_p)$ for all $p \in X$.

Problem 5. Let $f : X \to Y$ be a continuous map and $\mathcal{G}$ a sheaf on Y. Show that $(f^{-1}\mathcal{G})_p = \mathcal{G}_{f(p)}$ for all $p \in X$.

Problem 6. Let $f : X \to Y$ be a continuous map, $\mathcal{F}$ a sheaf on X, and $\mathcal{G}$ a sheaf on Y. Show that the map $Hom(\mathcal{G}, f_*\mathcal{F}) \to Hom(f^{-1}\mathcal{G}, \mathcal{F})$ constructed in class is bijective.