MAT536 HW3

Due 10/1 in class. Each problem is worth 10 points

Problem 1. Show that if R is a finitely generated reduced k-algebra then the space with functions Spec-m(R) is an affine variety.

Problem 2. Let X be any space with functions. A map $\phi : \mathbb{P}^n \to X$ is a morphism if and only if $\phi \circ \pi : \mathbb{A}^{n+1} \setminus \{0\} \to X$ is a morphism.

Problem 3. Prove that the Segre map $s : \mathbb{P}^n \times \mathbb{P}^m \to \mathbb{P}^N$ gives an isomorphism of $\mathbb{P}^n \times \mathbb{P}^m$ with a closed subvariety of $\mathbb{P}^N$, where $N = nm + n + m$.

Problem 4. Let $\phi : X \to Y$ be a morphism of spaces with functions and suppose $Y = \cup V_i$ is an open covering such that each restriction $\phi_i : \phi^{-1}(V_i) \to V_i$ is an isomorphism. Then ϕ is an isomorphism.

Problem 5. Asume that char $k \neq 2$. If $C = V_P(f) \subset \mathbb{P}^2$ is any curve defined by an irreducible homogeneous polynomial $f \in k[x, y, z]$ of degree 2, then $C \simeq \mathbb{P}^1$.

Problem 6. Let X and Y be spaces with functions and let (P, π_X, π_Y) and (P', π'_X, π'_Y) be two products of X and Y. Show that there is a unique isomorphism $\phi : P \to P'$ such that $\pi_X = \pi'_X \circ \phi$ and $\pi_Y = \pi'_Y \circ \phi$.