MAT 536 2020 Final Exam

Name:_____

I.D.:_____

1	2	3	4	5	TOTAL
10 pts	$10 \mathrm{~pts}$	$10 \mathrm{~pts}$	10 pts	$10 \mathrm{~pts}$	$50 \mathrm{~pts}$

1. (10 points) Let f(z) be an entire function. Given $a, b \in \mathbb{C}$, let C_R be the circle of radius R with center at the origin such that |a|, |b| < R. Evaluate the integral

$$\frac{1}{2\pi i} \int_{C_R} \frac{f(z)dz}{(z-a)(z-b)}.$$

2. (10 points) Suppose that the function f(z) is holomorphic in a domain D and C is a closed curve in D. Prove that

$$\int_C \overline{f(z)} f'(z) dz$$

is purely imaginary.

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3. (10 points) Determine whether there is a holomorphic function f(z) which maps the unit disk onto itself and satisfies $f(0) = \frac{1}{2}$ and $f'(0) = \frac{3}{4}$.

4. (10 points) Evaluate the integral

$$\int_0^\infty \frac{x \sin x}{x^2 + 1} dx.$$

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5. (10 points) Find the number of roots (counted with multiplicity) of the function $g(z) = 6z^3 + e^z + 1$ inside the unit disk (Hint: use Rouché's theorem).