## MAT 5362020 Final Exam

Name:
I.D.:

| 1 | 2 | 3 | 4 | 5 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 pts | 10 pts | 10 pts | 10 pts | 10 pts | 50 pts |
|  |  |  |  |  |  |

1. (10 points) Let $f(z)$ be an entire function. Given $a, b \in \mathbb{C}$, let $C_{R}$ be the circle of radius $R$ with center at the origin such that $|a|,|b|<R$. Evaluate the integral

$$
\frac{1}{2 \pi i} \int_{C_{R}} \frac{f(z) d z}{(z-a)(z-b)}
$$

2. (10 points) Suppose that the function $f(z)$ is holomorphic in a domain $D$ and $C$ is a closed curve in $D$. Prove that

$$
\int_{C} \overline{f(z)} f^{\prime}(z) d z
$$

is purely imaginary.
3. (10 points) Determine whether there is a holomorphic function $f(z)$ which maps the unit disk onto itself and satisfies $f(0)=\frac{1}{2}$ and $f^{\prime}(0)=\frac{3}{4}$.
4. (10 points) Evaluate the integral

$$
\int_{0}^{\infty} \frac{x \sin x}{x^{2}+1} d x
$$

5. (10 points) Find the number of roots (counted with multiplicity) of the function $g(z)=6 z^{3}+e^{z}+1$ inside the unit disk (Hint: use Rouché's theorem).
