MAT402 HW5

These are practice problems.

Problem 1. Show that the Lie group $\mathrm{SO}(n)$ is connected for all $n \in \mathbb{Z}^+$. Furthermore, show that $\mathrm{GL}(n, \mathbb{R})$ and $\mathrm{O}(n)$ each have exactly two connected components.

Problem 2. Show that both Lie groups $\mathrm{GL}(n, \mathbb{C})$ and $\mathrm{U}(n)$ are connected for all $n \in \mathbb{Z}^+$.

Problem 3. Prove that a connected Lie group G is abelian if and only if its Lie algebra $\mathfrak{g}$ is abelian.