

MAT402 HW4

These are practice problems.

Problem 1. Show that a pullback of a quotient bundle is a quotient of pullbacks. More precisely, let $f : M \rightarrow N$ be a smooth map and $V, V' \rightarrow N$ be vector bundles such that V' is a sub-bundle of V . Show that

$$f^*(V/V') \cong f^*(V)/f^*(V')$$

as vector bundles over M .

Problem 2. Let $\phi : V \rightarrow W$ be a smooth surjective vector-bundle homomorphism over a smooth manifold M . Show that

$$\text{Ker}\phi = \{v \in V : \phi(v) = 0\} \rightarrow M$$

is a sub-bundle of V .

Problem 3. Let $\pi : V \rightarrow M$ be a vector bundle. Show that there is an isomorphism

$$\Lambda^k(V^*) \rightarrow (\Lambda^k V)^*$$

of vector bundles over M .