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Introduction to Linear Algebra
MAT 211, Spring 2009

This is a bonus test to get extra 5 points towards the final score of 100. A complete solution is required for each problem. You will get 1 point for a correct solution of each problem. No partial credits.

Bonus Test

1. A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ has matrix $T_{\mathcal{A},\mathcal{B}} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ with respect to the bases $\mathcal{A} = \{(3,0,1),(1,1,0),(0,-2,1)\}$ in $\mathbb{R}^3$ and $\mathcal{B} = \{(0,-1),(1,4)\}$ in $\mathbb{R}^2$. Find the matrix of T with respect to the standard bases.

2. Let $T : \mathcal{P}_2 \to \mathcal{P}_2$ be a linear transformation defined by the formula $T(p) = p'' - 2xp' + 4p$.
a) Find the matrix of T with respect to the standard basis of $\mathcal{P}_2$.
b) Find the matrix of T with respect to the basis $\{1,\ 2x,\ 4x^2 - 2\}$ in $\mathcal{P}_2$.
c) Find bases in the kernel and image of T.
d) Verify the Kernel-Image theorem for T.
e) Is T an isomorphism? Explain!

3. Show that all symmetric 2×2 matrices form a subspace of the vector space M_2 of all square 2×2 matrices. Find a basis of this subspace. The same question for skew-symmetric matrices.

4. Let $\mathbf{u} = (x_1, y_1)$ and $\mathbf{v} = (x_2, y_2)$ be arbitrary vectors in $\mathbb{R}^2$. Show that the formula

$$< \mathbf{u}, \mathbf{v} >= (x_1, y_1) \cdot \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

defines an inner product in $\mathbb{R}^2$. In the inner product space with this inner product, for vectors $\mathbf{u} = (-1, 2)$ and $\mathbf{v} = (4, 2)$ do the following:
a) calculate $< \mathbf{u}, \mathbf{v} >$
b) find the norms of $\mathbf{u}$ and $\mathbf{v}$
c) find the distance between $\mathbf{u}$ and $\mathbf{v}$.

5. Let W be a subspace of $\mathbb{R}^4$ which is defined by

$$W = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 - x_2 + x_4 = 0,\ 2x_1 + x_2 - x_3 = 0\}.$$

Find the orthogonal complement $W^{\perp}$.