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Mathematics Department
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Proficiency Algebra
MAT 103, Fall 2013

Solutions to some problems from Practice Exam 1

Part C

Problem 1a. Calculate $\dfrac{\dfrac{3}{2}+\dfrac{2}{3}}{2\left(1-\dfrac{3}{7}\right)}$.

Solution.

$$\frac{\dfrac{3}{2}+\dfrac{2}{3}}{2\left(1-\dfrac{3}{7}\right)} = \frac{\dfrac{3\cdot 3+2\cdot 2}{2\cdot 3}}{2\cdot\dfrac{1\cdot 7-3}{7}} = \frac{\dfrac{9+4}{6}}{2\cdot\dfrac{4}{7}} = \frac{\dfrac{13}{6}}{\dfrac{8}{7}} = \frac{13}{6}\div\frac{8}{7} = \frac{13}{6}\cdot\frac{7}{8} = \frac{13\cdot 7}{6\cdot 8} = \boxed{\frac{91}{48}}.$$

Problem 2. What is the value of the expression $\dfrac{x+1}{x^2+1}$ for $x=-2$?

Solution.

$$\left.\frac{x+1}{x^2+1}\right|_{x=-2} = \frac{-2+1}{(-2)^2+1} = \frac{-1}{4+1} = \frac{-1}{5} = \boxed{-\frac{1}{5}}.$$

Problem 3. Clear parentheses and combine similar terms in the expression $(2x-1)(x-3)$.

Solution.

$$(2x-1)(x-3) = 2x\cdot x + 2x(-3) + (-1)x + (-1)(-3) = 2x^2 - 6x - x + 3 = 2x^2 - 7x + 3.$$

Problem 4b. Use the difference of squares formula to factor $4x^2-1$.

Solution. The expression to factor is a difference of squares

$$4x^2 - 1 = (2x)^2 - 1^2.$$

So we may use the difference of squares formula

$$a^2 - b^2 = (a-b)(a+b),$$

which is valid for **all** values of variables a and b. Taking $a=2x$ and $b=1$, we get

$$(2x)^2 - 1^2 = (2x-1)(2x+1).$$

Therefore,

$$4x^2 - 1 = (2x-1)(2x+1).$$

Problem 5. Factor the perfect square trinomial x^2-4x+4.

Solution. This problem is about applying a formula for the square of a sum/difference (or, a perfect square trinomial formula)

$$(x \pm y)^2 = x^2 \pm 2xy + y^2.$$

To use this formula, we rewrite the polynomial $x^2 - 4x + 4$:

$$x^2 - 4x + 4 = x^2 - 2x \cdot 2 + 2^2.$$

The square of a difference formula is valid for **any** values of variables x and y. Taking $y = 2$, we get

$$x^2 - 2x \cdot 2 + 2^2 = (x - 2)^2.$$

Problem 6. Simplify the following expressions

a) $\dfrac{x^2 - 4x + 4}{x^2 - 4}$ **b)** $\dfrac{x^2 - 4}{4x^2 - 8x}$

Solution. a) To simplify this fraction, we factor the numerator (a perfect square trinomial) and the denominator (a difference of squares):

$$\frac{x^2 - 4x + 4}{x^2 - 4} = \frac{(x-2)^2}{(x-2)(x+2)} = \frac{(x-2)(x-2)}{(x-2)(x+2)} = \frac{x-2}{x+2}.$$

b) To simplify the fraction, we factor both the numerator and denominator. The numerator is factored as a difference of squares $x^2 - 4 = (x-2)(x+2)$. The denominator is factored by pulling a common factor of $4x$ out of parentheses: $4x^2 - 8x = 4x(x-2)$. Therefore

$$\frac{x^2 - 4}{4x^2 - 8x} = \frac{(x-2)(x+2)}{4x(x-2)} = \frac{x+2}{4x}.$$

Problem 7. Solve the equations and check your solution by substitution into the equation:

b) $1 - 2(3x - 1) = 2x$ **c)** $\dfrac{2x}{3} + 4 = 2\left(x - \dfrac{2}{3}\right)$

Solution. b) We start with clearing the parentheses (distributing the factor of -2 over the parentheses):

$$1 - 2(3x - 1) = 2x \iff 1 - 6x + 2 = 2x.$$

Then we combine like terms of 1 and 2 and move $-6x$ to the right hand side of the equation

$$1 - 6x + 2 = 2x \iff 3 = 6x + 2x.$$

The rest is trivial:

$$3 = 6x + 2x \iff 3 = 8x \iff \frac{3}{8} = x.$$

So the solution is $x = {}^3/_8$. Check it by substitution into the equation:

$$1 - 2 \cdot \left(3 \cdot \frac{3}{8} - 1\right) = 2 \cdot \frac{3}{8} \qquad \text{Is it true?}$$

$$1 - 2 \cdot \left(\frac{9}{8} - 1\right) = \frac{3}{4}$$

$$1 - 2 \cdot \left(\frac{9-8}{8}\right) = \frac{3}{4}$$

$1 - 2 \cdot \left(\frac{1}{8}\right) = \frac{3}{4}$

$1 - \frac{1}{4} = \frac{3}{4}$

$\frac{3}{4} = \frac{3}{4}$ Yes, it is true! We have solved the equation correctly.

Answer: $x = {}^3\!/_8$.

c) Solve $\frac{2x}{3} + 4 = 2\left(x - \frac{2}{3}\right)$. Do you like to work with fractions? If not, multiply both sides of the equation by 3:

$$\frac{2x}{3} + 4 = 2\left(x - \frac{2}{3}\right) \iff 3 \cdot \left(\frac{2x}{3} + 4\right) = 3 \cdot 2\left(x - \frac{2}{3}\right)$$

and clear the parentheses

$$2x + 12 = 6x - 4.$$

Move $2x$ to the right and -4 to the left

$$12 + 4 = 6x - 2x.$$

From this

$$16 = 4x$$

$$x = 4.$$

Check the solution $x = 4$ by substitution into the equation:

$$\frac{2 \cdot 4}{3} + 4 = 2\left(4 - \frac{2}{3}\right) \qquad \text{Is it true?}$$

$\frac{8}{3} + 4 = 2 \cdot \frac{4 \cdot 3 - 2}{3}$

$\frac{8 + 4 \cdot 3}{3} = 2 \cdot \frac{10}{3}$

$\frac{20}{3} = \frac{20}{3}$ Yes, it is true! We have solved the equation correctly.

Answer: $x = 4$.

8. Solve the equation **c)** $\frac{\pi x}{2} + \sqrt{3} = x - \pi$.

Solution. You shouldn't be afraid of π and $\sqrt{3}$, they are just real numbers. The equation is linear and we solve it as usual. First, we collect all x-terms on the right hand side:

$$\frac{\pi x}{2} + \sqrt{3} = x - \pi \iff \sqrt{3} + \pi = x - \frac{\pi x}{2}.$$

Then combine like terms

$$\sqrt{3} + \pi = \left(1 - \frac{\pi}{2}\right) x,$$

and isolate x

$$\frac{\sqrt{3}+\pi}{1-\frac{\pi}{2}} = x.$$

This expression for x can be simplified

$$x = \frac{\sqrt{3}+\pi}{1-\frac{\pi}{2}} = \frac{\sqrt{3}+\pi}{\frac{2-\pi}{2}} = (\sqrt{3}+\pi) \div \frac{2-\pi}{2} = \boxed{\frac{2(\sqrt{3}+\pi)}{2-\pi}}.$$

9. Solve the equation $|3x-1| = \dfrac{4}{5}$ and check your solution by substitution.

Solution. First, we clear the absolute value:

$$|3x-1| = \frac{4}{5} \iff 3x-1 = \frac{4}{5} \qquad \text{or} \quad 3x-1 = -\frac{4}{5}.$$

Then, we solve separately each of two equations.

$$3x-1 = \frac{4}{5} \iff 3x = 1+\frac{4}{5} \iff 3x = \frac{5+4}{5} \iff 3x = \frac{9}{5} \iff x = \frac{9}{5\cdot 3} \iff x = \frac{3}{5}$$

$$3x-1 = -\frac{4}{5} \iff 3x = 1-\frac{4}{5} \iff 3x = \frac{5-4}{5} \iff 3x = \frac{1}{5} \iff x = \frac{1}{5\cdot 3} \iff x = \frac{1}{15}$$

So the equation has two solutions $x = \dfrac{3}{5}$ and $x = \dfrac{1}{15}$.

Don't forget to check **both** solutions:

$$\left|3\cdot\frac{3}{5}-1\right| = \frac{4}{5} \iff \left|\frac{9}{5}-1\right| = \frac{4}{5} \iff \left|\frac{9-5}{5}\right| = \frac{4}{5} \iff \left|\frac{4}{5}\right| = \frac{4}{5} \iff \frac{4}{5} = \frac{4}{5} \quad \text{True!}$$

$$\left|3\cdot\frac{1}{15}-1\right| = \frac{4}{5} \iff \left|\frac{3}{15}-1\right| = \frac{4}{5} \iff \left|\frac{1}{5}-1\right| = \frac{4}{5} \iff \left|\frac{1-5}{5}\right| = \frac{4}{5} \iff \left|\frac{-4}{5}\right| = \frac{4}{5} \quad \text{True!}$$

Answer: $x = \dfrac{3}{5}$ or $x = \dfrac{1}{15}$.

10. Solve the inequality $2x+3 \le -2(x-3)$ and give the answer in an interval notation.

Solution. Clear parentheses

$$2x+3 \le -2(x-3) \iff 2x+3 \le -2x+6$$

and collect x-terms on one side

$$2x+2x \le 6-3$$

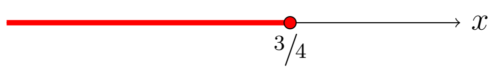
Simplify

$$4x \le 3$$

and get the solution

$$x \le \frac{3}{4}.$$

Draw a picture



And give the answer: $\left(-\infty, \dfrac{3}{4}\right]$.

11. Solve the problem using an equation: first, introduce a variable, then compose an equation, and finally, solve the equation. Solutions by other methods will give no credit.

b) Sales tax rate in Suffolk county is 8.625%. Tom spent \$1,303.50 in Stony Brook bookstore. What was the price for his purchase before tax?

Solution. Let x be the price for Tom's purchase **before** tax. (Observe that this is a natural choice for a variable, x is the quantity we have to find.) Then the price **after** tax is $x+0.0865x$, since $8.625\% = 0.08625$. Tom paid \$1,303.5 for his purchase, so $x + 0.08625x = 1,303.5$. This is a linear equation to solve: $1.08625x = 1303.5$, $\quad x = 1303.5 \div 1.08625$, $\ x = 1200$.

Answer: The price for Tom's purchase before tax was \$1,200.

f) In a triangle ABC, the angle A is 20° greater than the angle B and twice less as the angle C. Find the angles of the triangle.

Solution. Let x be the degree measure of the angle A. Then the angle B is of $(x-20)$ degrees, and the angle C is of $2x$ degrees. The sum of the angles in a triangle is 180°, therefore

$$x + (x - 20) + 2x = 180.$$

Let us solve the equation:

$$x+(x-20)+2x = 180 \iff 4x-20 = 180 \iff 4x = 180+20 \iff 4x = 200 \iff x = 50.$$

So $A = 50^\circ$, $\ B = 50 - 20 = 30^\circ$ and $C = 2 \cdot 50 = 100^\circ$.

Check (it is not required by the problem, but let's be on a safe side): The angle $A = 50^\circ$ is indeed 20° greater than the angle $B = 30^\circ$, and twice less as the angle $C = 100^\circ$. Moreover,

$$50^\circ + 30^\circ + 100^\circ = 180^\circ.$$

Therefore, the solution is correct.

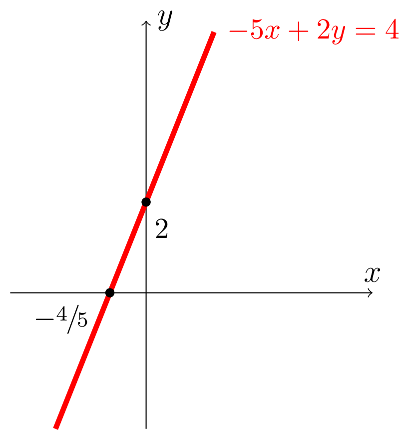
Answer: The angles are 50°, 30° and 100°.

12. Draw the graphs of the following equations:

a) $-5x + 2y = 4$ **b)** $x = 3$ **c)** $y = -\pi$ **d)** $x = 0$, **e)** $y = 0$.

Solution. In general, to draw a line from its equation, it is enough to plot **two** different points belonging to the line and draw a line passing through them. You can choose **any** two points, but choosing the intercepts point(s) gives rise to simpler calculations.

a) The equation $-5x + 2y = 4$ is linear, therefore, its graph is a straight line. To draw a line, we need two points on it. Which ones? **Any** two points. Thinking about simplicity of calculations, let's take $x = 0$, then $-5 \cdot 0 + 2y = 4$, that is $y = 2$. So the point $(0, 2)$ belong to the line. Take now $y = 0$. Then $-5x + 2 \cdot 0 = 4$, that is $x = -4/5$. So the point $(-4/5, 0)$ belong to the line. Place these two points on the plane a draw a line through them:

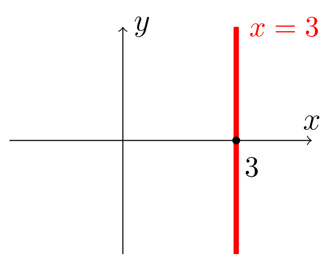


Alternatively, we may rewrite the equation of the line in the **two intercept** form

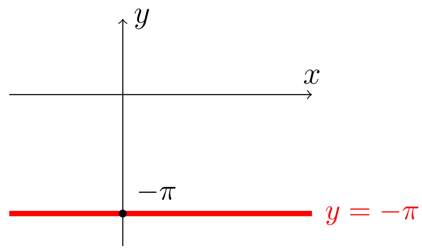
$$-5x + 2y = 4 \iff \frac{x}{-4/5} + \frac{y}{2} = 1.$$

This form of the equation gives us the intercepts right away: x-intercept is $-4/5$ and y-intercept is 2.

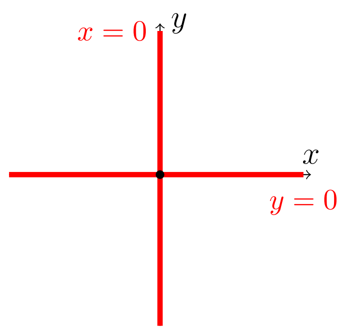
b) The equation $x = 3$ represent a **vertical** line passing through $(3, 0)$:



c) The equation $y = -\pi$ represents a **horizontal** line passing through $(0, -\pi)$:



d), **e)** The equations $x = 0$ describe a vertical line passing through $(0, 0)$, that is, the y-axis. The equations $y = 0$ describe a horizontal line passing through $(0, 0)$, that is, the x-axis.



Remark: Here are several hints how to produce a nice looking picture which can please anybody (including yourself and a grader).

• Draw x- and y- axis **perpendicular** to each other.

• On the axes, label **only** the points you need (for example, the intercepts). Unnecessary labels make the drawing busy (see the textbook for examples).

• Make sure that your line is a **straight** line indeed. Write down the equation of the line a bit aside the line.

Part AB

Problem 2. Clear parentheses in the expression $x(x(x+1)-1)+1$.

Solution. We clear the parentheses step by step, starting from the **inner** parentheses:

$$x(x(x+1)-1)+1 = x((x^2+x)-1))+1 = x(x^2+x-1)+1 = x^3+x^2-x+1.$$

Problem 4. Solve the inequalities and give the answer in an interval notation

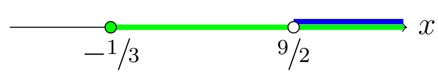
a) $2(2-x) \le x+5 < 3x-4$ **b)** $1-x \le 2x+1 < 4$.

Solution for a). The double inequality is equivalent to two inequalities which should be satisfied simultaneously: $2(2-x) \le x+5$ and $x+5 < 3x-4$.

$$2(2-x) \le x+5 \iff 4-2x \le x+5 \iff -3x \le 1 \iff x \ge -\frac{1}{3}$$

$$x+5 < 3x-4 \iff 9 < 2x \iff \frac{9}{2} < x \iff x > \frac{9}{2}$$

The solution of the double inequality consists of all real numbers x for which $x \ge -\frac{1}{3}$ **and** $x > \frac{9}{2}$. It is the **intersection** of the intervals $[-1/3, \infty)$ and $(9/2, \infty)$:



The intersection (the common part) of $[-1/3, \infty)$ and $(9/2, \infty)$ is

$$[-1/3, \infty) \cap (9/2, \infty) = (9/2, \infty).$$

Answer: $(9/2, \infty)$.

Solution for b). $1-x \le 2x+1 < 4$ is a double inequality. To solve it, we have to find all values of x for which **both** inequalities involved are satisfied:

$1-x \le 2x+1$ **and** $2x+1 < 4$.

$$1-x \le 2x+1 \iff 0 \le 3x \iff x \ge 0.$$

$$2x+1 < 4 \iff 2x < 3 \iff x < 3/2.$$

The solution of the double inequality is the **intersection** between the obtained intervals (their common, or overlapping, part):

$$[0, \infty) \cap (-\infty, 3/2) = [0, 3/2).$$

Answer: $[0, 3/2)$

Problem 5. For the line $-2x + 3y = 6$, find the slope, the x-intercept and the y-intercept. Draw the line on the coordinate system. Show on your picture the x- and y-intercepts.

Solution. To find the slope and the y-intercept, we have to rewrite the equation in the **slope-intercept** form

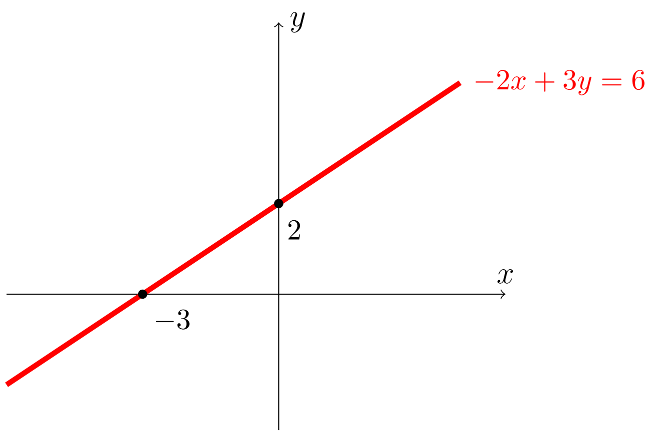
$$-2x + 3y = 6 \iff 3y = 2x + 6 \iff y = \frac{2}{3}x + 2.$$

As we see from the latter equation, the $\boxed{\text{slope is } 2/3}$ and the $\boxed{y\text{-intercept is } 2}$.

The x-intercept is a point where the line intersect the x-axis. At this point, $y = 0$. So we plug in $y = 0$ into the equation and get the x-value:

$$-2x + 3 \cdot 0 = 6 \implies -2x = 6 \implies x = -3.$$

Therefore, the $\boxed{x\text{-intercept is } -3}$.



Problem 7. The Earth's equator is 24,900 miles long. Using this fact, find the radius of the Earth.

Solution. The equator is a circle. Its circumference is 24,900 miles. We have to find the radius of the circle. The relation between the radius R of a circle and the circle circumference C is given by the formula $C = 2\pi R$. Solving this equation for R, we get $R = \dfrac{C}{2\pi}$.

The radius of the Earth is, therefore,

$$R = \frac{C}{2\pi} = \frac{24,900}{2\pi} \approx \frac{24,900}{2 \cdot 3.14} = \frac{24,900}{6.28} = \boxed{3,990 \text{ miles}}.$$

Remark. The actual radius of the Earth is $3,959$ miles.