

Centers of Simply Connected Semisimple Groups (via Coroot Parameterization)

$\mathbf{SL}_{n+1}$ A_n $\bullet \xrightarrow{t^1} \bullet \xrightarrow{t^2} \bullet \xrightarrow{t^3} \dots \xrightarrow{t^{n-2}} \bullet \xrightarrow{t^{n-1}} \bullet \xrightarrow{t^n} \bullet$ $t^{n+1} = 1$

$\mathbf{Spin}_{2n+1}$ $B_n \ (n \geq 2)$ $\bullet \xrightarrow{1} \bullet \xrightarrow{1} \bullet \xrightarrow{\dots} \bullet \xrightarrow{1} \bullet \xrightarrow{1} \bullet \xrightarrow{\zeta} \bullet$ $\zeta^2 = 1$

$\downarrow$

$\mathbf{SO}_{2n+1}$ $B_1 = A_1$ $1 \quad 1 \quad 1 \quad \dots \quad 1 \quad 1 \quad \zeta$

$\mathbf{Sp}_{2n}$ $C_n \ (n \geq 2)$ $\bullet \xrightarrow{\zeta} \bullet \xrightarrow{1} \bullet \xrightarrow{\zeta} \dots \xrightarrow{\zeta^{n-2}} \bullet \xrightarrow{\zeta^{n-1}} \bullet \xrightarrow{\zeta^n} \bullet$ $\zeta^2 = 1$

$C_1 = A_1$

$C_2 = B_2$

$\mathbf{Spin}_{2n}$ $D_n \ (n \geq 4)$ $\bullet \xrightarrow{\zeta^2} \bullet \xrightarrow{1} \bullet \xrightarrow{\zeta^2} \dots \xrightarrow{1} \bullet \xrightarrow{\zeta^2} \bullet \begin{matrix} \nearrow \zeta^1 \\ \searrow \zeta^{-1} \end{matrix}$ $\zeta^4 = 1$

$\downarrow$

$\mathbf{SO}_{2n}$ $(n \text{ odd})$ $\zeta^2 \quad 1 \quad \zeta^2 \quad \dots \quad 1 \quad \zeta^2$ “ $\zeta^2 = -1$ ”

$D_3 = A_3$

not adjoint

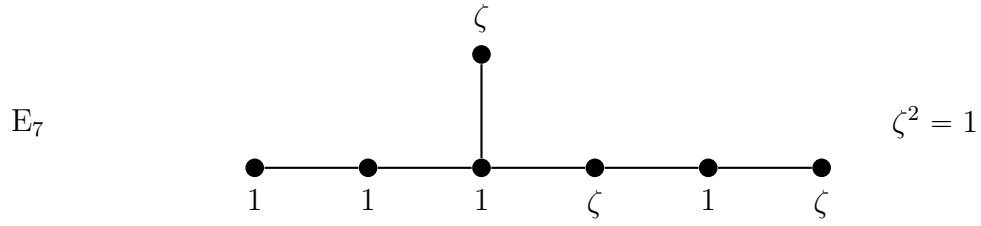
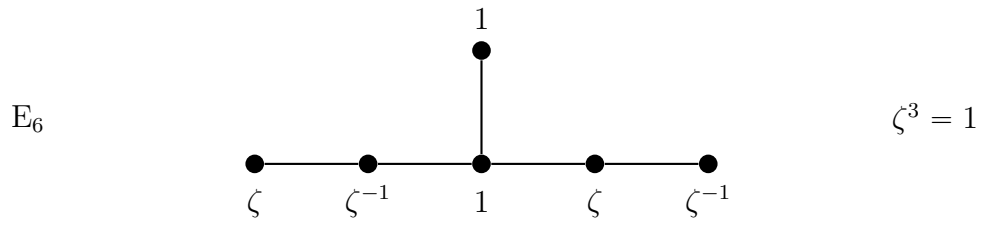
$\mathbf{Spin}_{2n}$ $D_n \ (n \geq 4)$ $\bullet \xrightarrow{tt'} \bullet \xrightarrow{1} \bullet \xrightarrow{tt'} \dots \xrightarrow{tt'} \bullet \xrightarrow{1} \bullet \begin{matrix} \nearrow t \\ \searrow t' \end{matrix}$ $(t, t') \in \mu_2 \times \mu_2$

$\downarrow$

$\mathbf{SO}_{2n}$ $(n \text{ even})$ $tt' \quad 1 \quad tt' \quad \dots \quad tt' \quad 1$

$D_2 = A_1 \times A_1$

not adjoint



E_8, F_4, G_2 : trivial center

$$\mathbf{SO}_6 \cong \mathbf{SL}_4/\mu_2 \quad (D_3 = A_3)$$

$$\mathbf{SO}_5 \cong \mathbf{Sp}_4/\text{center} \quad (B_2 = C_2)$$

$$\mathbf{SO}_4 \cong (\mathbf{SL}_2 \times \mathbf{SL}_2)/\Delta(\mu_2) \quad (D_2 = A_1 \times A_1)$$

$$\mathbf{SO}_3 \cong \mathbf{PGL}_2 \quad (B_1 = A_1)$$

$$\mathbf{SO}_2 \cong \mathbf{G}_m$$