Centers of Simply Connected Semisimple Groups (via Coroot Parameterization)

$\mathbf{SL}_{n+1}$ $\quad$ A_n

$t^{n+1} = 1$

$\mathbf{Spin}_{2n+1}$ $\quad$ B_n $(n \geqslant 2)$

$\downarrow$

$\mathbf{SO}_{2n+1}$ $\quad$ $\mathrm{B}_1 = \mathrm{A}_1$

$\zeta^2 = 1$

$\mathbf{Sp}_{2n}$ $\quad$ C_n $(n \geqslant 2)$

$\mathrm{C}_1 = \mathrm{A}_1$

$\mathrm{C}_2 = \mathrm{B}_2$

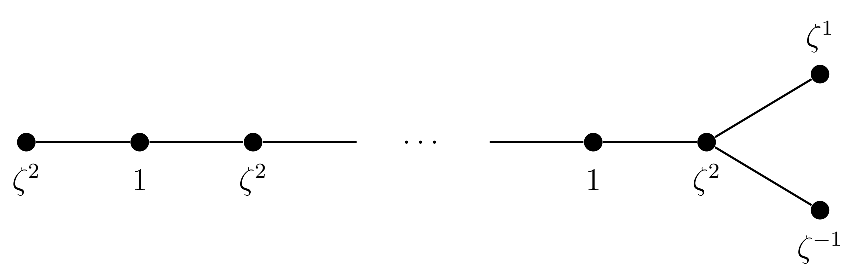
$\zeta^2 = 1$

$\mathbf{Spin}_{2n}$ $\quad$ D_n $(n \geqslant 4)$

$\downarrow$

$\mathbf{SO}_{2n}$ $\quad$ (n odd)

not adjoint $\quad$ $\mathrm{D}_3 = \mathrm{A}_3$



$\zeta^4 = 1$

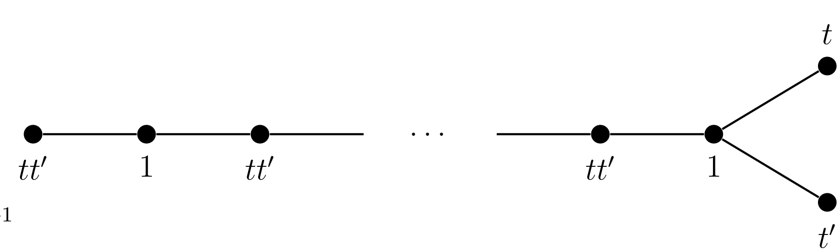
"$\zeta^2 = -1$"

$\mathbf{Spin}_{2n}$ $\quad$ D_n $(n \geqslant 4)$

$\downarrow$

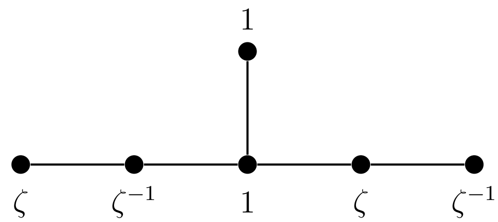
$\mathbf{SO}_{2n}$ $\quad$ (n even)

not adjoint $\quad$ $\mathrm{D}_2 = \mathrm{A}_1 \times \mathrm{A}_1$



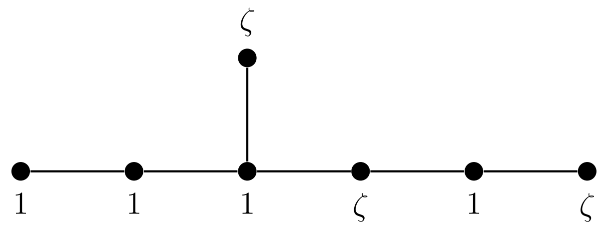
$(t, t') \in \mu_2 \times \mu_2$

E_6



$\zeta^3 = 1$

E_7



$\zeta^2 = 1$

$$E_8, \;\; F_4, \;\; G_2 : \text{trivial center}$$

$$\mathbf{SO}_6 \cong \mathbf{SL}_4/\mu_2 \quad (\; D_3 = A_3 \;)$$

$$\mathbf{SO}_5 \cong \mathbf{Sp}_4/\text{center} \quad (\; B_2 = C_2 \;)$$

$$\mathbf{SO}_4 \cong (\mathbf{SL}_2 \times \mathbf{SL}_2)/\Delta(\mu_2) \quad (\; D_2 = A_1 \times A_1 \;)$$

$$\mathbf{SO}_3 \cong \mathbf{PGL}_2 \quad (\; B_1 = A_1 \;)$$

$$\mathbf{SO}_2 \cong \mathbf{G}_m$$